Computer Project # 1
Nonlinear Springs

Goal: Investigate the behavior of nonlinear springs.

Tools needed: ode45, plot routines.

Description: Sometimes for certain (nonlinear) spring-mass systems, the spring force is not given by Hooke’s Law but instead satisfies

\[ F_{\text{Spring}} = ku + \epsilon u^3 \]

where \( k > 0 \) is the spring constant and \( \epsilon \) is small but may be positive or negative and represents the “strength” of the spring (\( \epsilon = 0 \) gives Hooke’s Law). The spring is called a hardening spring if \( \epsilon > 0 \) and a softening spring if \( \epsilon < 0 \).

Questions: Suppose a nonlinear spring-mass system satisfies the initial value problem

\[
\begin{align*}
    u'' + u + \epsilon u^3 &= 0 \\
    u(0) &= 0, \quad u'(0) = 1
\end{align*}
\]

(1) Let \( \epsilon = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0 \) and plot the solutions of the above initial value problem for \( 0 \leq t \leq 15 \). Estimate the maximum amplitude of the spring. What appears to happen to the amplitude as \( \epsilon \) increases? Let \( T_1 \) = first time the mass reaches equilibrium after \( t = 0 \). Estimate \( T_1 \) when \( \epsilon = 0.0, 0.2, 0.4, 0.6, 1.0 \). What appears to happen to \( T_1 \) as \( \epsilon \) increases?

(2) Let \( \epsilon = -0.1, -0.2, -0.3, -0.4 \) and plot the solutions of the above initial value problem for \( 0 \leq t \leq 15 \). Estimate the maximum amplitude of the spring. What appears to happen to the amplitude as \( \epsilon \) decreases? Let \( T_1 \) = first time the mass reaches equilibrium after \( t = 0 \). Estimate \( T_1 \) when \( \epsilon = -0.1, -0.2, -0.3, -0.4 \). What appears to happen this time to \( T_1 \) as \( \epsilon \) decreases?