Supplementary Problems # 2

1. **Page 44**: # 4, 5(c)(d)(e), 6.

2. Sketch the vector fields corresponding to these complex functions

   (a) \( f(z) = \frac{z}{|z|} \)  
   (b) \( g(z) = -iz \)  
   (c) \( h(z) = \frac{\bar{z}}{|z|} \)

3. **Page 45**: # 13 (Electric fields and equilibrium points)

4. **Page 49**: # 5, 9(d)(f).

5. **Page 57**: # 14, 15.

6. **Page 62**: # 1(c), 6, 15

7. The \( z \) and \( \bar{z} \) derivatives of a function \( f(z) = u(x, y) + iv(x, y) \) are defined as follows:

   \[
   \frac{\partial f}{\partial z} \equiv \frac{1}{2} \left( \frac{\partial f}{\partial x} - i \frac{\partial f}{\partial y} \right)
   \]

   \[
   \frac{\partial f}{\partial \bar{z}} \equiv \frac{1}{2} \left( \frac{\partial f}{\partial x} + i \frac{\partial f}{\partial y} \right)
   \]

   (a) Show that the CR equations are equivalent to the single equation \( \frac{\partial f}{\partial \bar{z}} = 0 \).

   (b) Show that if \( H(x, y) \) is real-valued and \( H_{xy} = H_{yx} \), then \( \Delta H = 4 \frac{\partial^2 H}{\partial z \partial \bar{z}} \).

   (c) Find a real-valued solution \( H(x, y) \) to the Poisson Equation \( \Delta H(x, y) = 16x \).

8. **Page 68**: # 3(c), 10.

9. Find a harmonic conjugate for \( \phi(x, y) = \Re\{z^{10}\} \).

10. Find a harmonic conjugate for \( \psi(x, y) = \Im\{z^{10}\} \).

11. Sketch level curves for \( u \) and \( v \) corresponding to the function \( f(z) = z^2 - 2z \).