Matlab Tutorial

Introductory Comments

- To access MATLAB locally:
  
  \[ \text{Start} \rightarrow \text{All Programs} \rightarrow \text{Standard Software} \rightarrow \]
  
  \[ \text{Computational Packages} \rightarrow \text{MATLAB6.5.1} \rightarrow \text{MATLAB6.5.1} \]

- Type \texttt{quit} or \texttt{exit} to close MATLAB.
- MATLAB is case sensitive so \texttt{a} and \texttt{A} are different.
- Comments may be added after a “%”.
- To clear the contents of \texttt{A}, type \texttt{clear A}; while to clear all variables type \texttt{clear}.
- To save your session type \texttt{save mysession}; to recover it later type \texttt{load mysession}.
- Initially the letters \texttt{i} and \texttt{j} denote the complex constant $\sqrt{-1}$, while \texttt{pi} denotes $\pi$.
  (Complex numbers are entered for example as $2 - 4i$, $2 - 4 * i$ or \texttt{complex(2,-4)}.)
- If no printing is wanted, end line with semicolon “;”.
- Multiple expressions may be on a single line if separated by commas (or by semicolons if you want to suppress printing).
- The expression \texttt{ans} means “current answer”.
- The expression \texttt{NaN} means “not a number”.
- A few typing shortcuts

<table>
<thead>
<tr>
<th>Key</th>
<th>Operation</th>
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</thead>
<tbody>
<tr>
<td>Up Arrow</td>
<td>Recall previous line</td>
</tr>
<tr>
<td>Down Arrow</td>
<td>Recall next line</td>
</tr>
<tr>
<td>Home</td>
<td>Beginning of line</td>
</tr>
<tr>
<td>End</td>
<td>End of line</td>
</tr>
<tr>
<td>Esc</td>
<td>Clear line</td>
</tr>
</tbody>
</table>

(1) Inputting Data

(a) To input a number say, 23.46, type

\[ \texttt{>> x= 23.46} \]

results in

\[ \texttt{x = 23.56} \]

To assign \texttt{x} the value 23.46 and suppress printing, use semicolon \texttt{>> x= 23.46;}

(b) Row vectors (separate entries by spaces or commas):

\[ \texttt{>> R=[1 3 -8]} \]

results in

\[ \texttt{R = 1 3 -8} \]
(c) Column vectors (entries separated by semicolons):

```matlab
>> C=[4; 1; -2]
results in
C =
  4
  1
 -2
```

(d) To input a matrix, say \( A = \begin{bmatrix} 1 & 2 & 3 \\ 8 & -1 & 0 \end{bmatrix} \), type

```matlab
>> A=[1 2 3; 8, -1, 0]  % Columns are separated by semicolons “;”
results in
A =
  1   2   3
  8  -1   0
```

(e) To view (or modify) a specific entry, say \( A(2, 1) \), type

```matlab
>> A(2,1)
results in
ans =
  8
```

If we wish to change that entry to the value say 13, then type

```matlab
>> A(2,1)=13
results in
A =
  1   2   3
  13  -1   0
```

To view the 2\textsuperscript{nd} row of the matrix \( A \), type:

```matlab
>> A(2,:)
% Don’t forget the comma
results in
ans =
 13   -1   0
```

To replace the 2\textsuperscript{nd} row of the above matrix \( A \) by the above row vector \( R=[1 3 -8] \), type

```matlab
>> A(2,:)=R
results in
A =
  1   2   3
 1   3  -8
```

Similarly to view the 3\textsuperscript{rd} column of a matrix \( A \), type:

```matlab
>> A(:,3)
% Don’t forget the comma
and to replace the 3\textsuperscript{rd} column of \( A \) by a column vector \( C \), type
>> A(:,3)=C
```

(f) Augmented matrices. To obtain the augmented matrix \( [A|b] \), type:

```matlab
>> C=[A,b];
```
(g) The outputs can be of several formats. The most commonly used are:

- `format short`  % displays numbers to 4 decimal places
- `format long`  % displays numbers to 14 decimal places
- `format rat`  % displays numbers in rational form \( \frac{p}{q} \)

For example,

```
>> format rat % all numbers after this will be displayed as \( \frac{p}{q} \)
For example, type
>> a=12.33
results in

\[ a = \frac{1233}{100} \]
```

(2) **Special Matrices**

(a) Identity matrix. For example, to obtain the $3 \times 3$ identity matrix $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, type:

```
>> I=eye(3)
results in

\[ I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \]
```

To obtain say a $2 \times 3$ identity matrix, type

```
>> J=eye(2,3)
results in

\[ J = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \]
```

(b) To obtain an $m \times n$ matrix of zeros (or ones):

```
>> K=zeros(2,3)
results in

\[ K = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \]
```

while typing

```
>> M=ones(2,3)
results in

\[ M = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \]
```

(c) Transpose. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 13 & -1 & 0 \end{bmatrix}$, the transpose is

```
>> B=A’  % or type B=transpose(A)
results in
```
\[ B = \begin{bmatrix} 1 & 13 \\ 2 & -1 \\ 3 & 0 \end{bmatrix} \]

(d) Diagonal matrices.

\[ \text{>> diag([3 0 -5])} \]

\[ \text{results in} \]

\[ \text{ans} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -5 \end{bmatrix} \]

To display the diagonal elements of an arbitrary matrix \( A \), type

\[ \text{>> diag(A)} \]

(3) Matrix Operations

There are two types of operations on matrices, standard and entry-by-entry:

<table>
<thead>
<tr>
<th>Operation</th>
<th>Standard</th>
<th>Entry-by-entry</th>
</tr>
</thead>
<tbody>
<tr>
<td>addition</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>subtraction</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>multiplication</td>
<td>*</td>
<td>.*</td>
</tr>
<tr>
<td>left division</td>
<td>\</td>
<td>\</td>
</tr>
<tr>
<td>right division</td>
<td>/</td>
<td>./</td>
</tr>
<tr>
<td>exponentiation</td>
<td>^</td>
<td>./</td>
</tr>
</tbody>
</table>

For example if \( A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix} \) and \( B = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \), then \( AB = \begin{bmatrix} 5 & 11 \\ -1 & -3 \end{bmatrix} \).

The entry-by-entry multiplication (Hadamard product) of \( A \) and \( B \) is \( \begin{bmatrix} 1 & 6 \\ -2 & 0 \end{bmatrix} \).

To obtain these we type:

\[ \text{>> M=A*B} \]

\[ \text{results in} \]

\[ M = \begin{bmatrix} 5 & 11 \\ -1 & 3 \end{bmatrix} \]

and

\[ \text{>> M=A.*B} \]

\[ \text{results in} \]

\[ M = \begin{bmatrix} 1 & 6 \\ -2 & 0 \end{bmatrix} \]
(a) To add two matrices type
>> C=A+B;

(b) Scalar multiplication:
>> C=3*A;  % Each entry of A is multiplied by 3

(c) Left division $A\backslash B$ means $\frac{B}{A}$ (i.e. $A^{-1}B$). This is useful for solving linear systems $Ax = b$, the solution being $x = A\backslash b$ (see (4)(c), below).

(d) Right division $A/B$ means $\frac{A}{B}$ (i.e., $AB^{-1}$)

(e) Reduced row echelon form. To find the row-reduced echelon form of $A = \begin{bmatrix} 1 & -1 & 2 & -2 \\ 2 & 1 & 0 & 3 \\ 2 & 3 & 2 & 0 \end{bmatrix}$,

    type:
    >> A=[1 -1 2 -2; 2 1 0 3; 2 3 2 0]; format rat;  % so answer will be rational numbers
    >> rref(A)
    results in 
    \[
    \text{ans} =
    \begin{bmatrix}
    1 & 0 & 0 & 10/7 \\
    0 & 1 & 0 & 1/7 \\
    0 & 0 & 1 & -23/14 \\
    \end{bmatrix}
    \]

(f) Rank of a matrix:
>> rank(A);

(g) Size of a matrix. If $A$ is $3 \times 2$ then type
>> size(A)

    results in 
    \[
    \text{ans} =
    \begin{bmatrix}
    3 & 2 \\
    \end{bmatrix}
    \]

(h) Determinant of a square matrix $A$:
>> det(A);

(i) Inverse of a square matrix $A$:
>> inv(A);

(j) Eigenvalues and eigenvectors. To find eigenvalues of say the matrix $A = \begin{bmatrix} 1 & -4 & -1 \\ 3 & 2 & 3 \\ 1 & 1 & 3 \end{bmatrix}$,

    type
    >> A=[1 -4 -1; 3 2 3; 1 1 3];
    >> eig(A)
    results in 
    \[
    \text{ans} =
    \begin{bmatrix}
    2 + 3i \\
    2 - 3i \\
    2 \\
    \end{bmatrix}
    \]

To obtain eigenvalues and eigenvectors of $A$, type
\[ [V, D] = \text{eig}(A) \] % Eigenvectors are column vectors of V, the eigenvalues are the diagonal elements of D

\[ results\ in\ 
V = 
\begin{array}{ccc}
0.7792 & 0.7792 & -0.7071 \\
-0.1375 - 0.5500i & -0.1375 + 0.5500i & 0.0000 \\
-0.2292 - 0.1375i & -0.2292 + 0.1375i & 0.7071 \\
\end{array}
\]

\[ D = 
\begin{array}{ccc}
2.0000 + 3.0000i & 0 & 0 \\
0 & 2.0000 - 3.0000i & 0 \\
0 & 0 & 2.0000 \\
\end{array}
\]

(4) Solving Linear Systems \( A\vec{x} = \vec{b} \)

To illustrate the different techniques we will use same example

\[
\begin{bmatrix}
0 & 2 & 1 \\
3 & 5 & -5 \\
2 & 4 & -2
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
= 
\begin{bmatrix}
-2 \\
1 \\
2
\end{bmatrix}
\]

Begin by inputting the matrices \( A \) and \( b \):
\[ >> A=[0 \ 2 \ 1; \ 3 \ 5 \ -5; \ 2 \ 4 \ -2]; \ b=[-2; \ 1; \ 2]; \]

(a) *Gaussian Elimination/Row-Reduction.* The solution is obtained by row-reducing the augmented matrix \([A|b]\) (see (1)(f) above for augmented matrix). Type
\[ >> \text{rref}([A,b]) \]
\[ results\ in\ 
\text{ans} = 
\begin{array}{ccc}
1 & 0 & 0 & 7 \\
0 & 1 & 0 & -2 \\
0 & 0 & 1 & 2 \\
\end{array}
\]
Hence \( x_1 = 7, \ x_2 = -2, \ x_3 = 2 \)

(b) *Inverses.* The solution is given by \( \vec{x} = A^{-1}\vec{b} \). Type
\[ >> x=\text{inv}(A)*b \]
\[ results\ in\ 
x = 
\begin{array}{c}
7.000 \\
-2.000 \\
2.000
\end{array}
\]

(c) *Left multiplication.* Solution is given by \( x = A\backslash b \). Type
\[ >> x=A\backslash b \]
\[ results\ in\ 
x = 
\begin{array}{c}
7.000 \\
-2.000 \\
2.000
\end{array}
\]
(d) *Cramer’s Rule.* Solution is given by $x_k = \frac{\det A_k}{\det A}$, where $A_k$ is $A$ with the $k^{th}$ column replaced by $\bar{b}$. To find for example $x_3$ in above example, type

```
>> A3=A; A3(:,3)=b;  % don’t want to change the original matrix A
>> x3=det(A3)/det(A)
```

results in

```
x3 =
    2
```

(5) More tutorials - Check this website:

http://amath.colorado.edu/computing/Matlab/tutorials.html