10 Find a basis for the submodule of $\mathbb{Z}^{(3)}$ generated by $(1, 0, -1), (2, -3, 1), (0, 3, 1), (3, 1, 5)$.

10 Over the polynomial ring $\mathbb{Q}[x]$ find a base for the submodule of $\mathbb{Q}[x]^{(3)}$ generated by $(2x-1, x, x^2+3), (x, x, x^2)$, and $(x+1, 2x, 2x^2-3)$.

10 Find a basis for the $\mathbb{Z}$-submodule of $\mathbb{Z}^{(3)}$ consisting of the set of all $(x, y, z)$ satisfying $x + 2y + 3z = 0$ and $x + 4y + 9z = 0$.

10 Put this matrix into normal form:

$$
\begin{bmatrix}
6 & 2 & 3 & 0 \\
2 & 3 & -4 & 1 \\
-3 & 3 & 1 & 2 \\
-1 & 2 & -3 & 5
\end{bmatrix}
$$

10 Determine the structure of $M = \mathbb{Z}^{(3)}/K$ where $K$ is generated by $f_1 = (2, 1, -3)$ and $f_2 = (1, -1, 2)$.

10 Let $D = \mathbb{R}[X]$ and assume that $M$ is a direct sum of cyclic $D$-modules whose order ideals (annihilating ideals) are the ideals generated by the polynomials $(x - 1)^3, (x^2 + 1)^2, (x - 1)(x^2 + 1)^4, (x + 2)(x^2 + 1)^2$. Determine the elementary divisors of $M$ and the invariant factors of $M$.

10 If $N$ is a direct summand of $M$ (i.e. $M = N \oplus K$, show $N$ is pure in $M$.

10 If $N$ is a pure submodule of $M$ and $\text{ann}(x + N) = (d)$, prove that $w$ can be chosen in $x + N$ such that $w + N = x + N$ and $\text{ann}(w) = (d)$.

10 If $N$ is a pure submodule of a finitely generated module $M$ over a PID $D$, prove that $N$ is a direct summand of $M$. You may assume that a finitely generated module of a PID is a direct product of cyclic modules.