MA 265 Lecture 15

Section 4.3 Subspaces (cont)

Definition of Linear Combination
Let \( v_1, v_2, \ldots, v_k \) be vectors in a vector space \( V \).

Example 1. Every polynomial of degree \( \leq 2 \) is a linear combination of \( t^2, t, 1 \).

Example 2. Show that the set of all vectors in \( \mathbb{R}^3 \) of the form \(
\begin{bmatrix}
    a \\
    b \\
    a + b
\end{bmatrix}
\) is a linear combination of \( v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \) and \( v_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \).
Example 3. In $\mathbb{R}^3$, let

\[ \mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \]

Verify that the vector

\[ \mathbf{v} = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix} \]

is a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$. 
Example 4. Consider the homogeneous system

$$Ax = 0$$

where $A$ is an $m \times n$ matrix. The set $W$ of solutions is a subset of $\mathbb{R}^n$. Verify that $W$ is a subspace of $\mathbb{R}^n$ (called solution space).

Remark The set of all solutions of the linear system $Ax = b$, with $b \neq 0$, is