Section 4.9  Rank of a Matrix

Definition  Let 

\[
A = \begin{bmatrix}
  a_{11} & a_{12} & \cdots & a_{1n} \\
  a_{21} & a_{22} & \cdots & a_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  a_{m1} & a_{m2} & \cdots & a_{mn}
\end{bmatrix}
\]

be an \( m \times n \) matrix.

- The rows of \( A \),

- The columns of \( A \),

Remark  If \( A \) and \( B \) are row equivalent matrices, then
We can use this remark to find a basis for a subspace spanned by a given set of vectors.

**Example 1.** Find a basis for the subspace $V$ of $\mathbb{R}_5$ that is spanned by $S = \{v_1, v_2, v_3, v_4\}$ where

$$v_1 = [1, -2, 0, 3, -4], \quad v_2 = [3, 2, 8, 1, 4], \quad v_3 = [2, 3, 7, 2, 3], \quad v_4 = [-1, 2, 0, 4, 3],$$

**Example 2.** Let $V$ be the subspace of Example 1. Given that the vector $v = [5, 4, 14, 6, 3]$ is in $V$, write $v$ as a linear combinations of the base determined by Example 1.
**Definition** The dimension of the row (column) space of $A$ is called

**Remark** If $A$ and $B$ are row equivalent,

**Example 3.** *Compute the row rank of $A$ given by*

\[
A = \begin{bmatrix}
1 & -2 & 0 & 3 & -4 \\
3 & 2 & 8 & 1 & 4 \\
2 & 3 & 7 & 2 & 3 \\
-1 & 2 & 0 & 4 & -3 \\
\end{bmatrix}
\]
Example 4. Compute the column rank of $A$ in Example 3.

**Theorem** Let $A$ be an $m \times n$ matrix. Then