Section 5.1  Length and Direction in $\mathbb{R}^2$ and $\mathbb{R}^3$

Length of Vectors in $\mathbb{R}^2$

Let $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ be a vector in $\mathbb{R}^2$. The _________ or _________ of $\mathbf{v}$, denoted by $\|\mathbf{v}\|$, is

The distance between vectors $\mathbf{u}$ and $\mathbf{v}$ is defined as

Length of Vectors in $\mathbb{R}^3$

Let $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$ be a vector in $\mathbb{R}^3$. The length of $\mathbf{v}$ is defined as

The distance between vectors $\mathbf{u}$ and $\mathbf{v}$ is defined as
Direction

We consider the angle $\theta$, $0 \leq \theta \leq \pi$ between two vectors.

In $\mathbb{R}^2$, we plot the angle of two vectors $u$ and $v$:

By law of cosines:

we have

Similarly, if $u$ and $v$ are vectors in $\mathbb{R}^3$, the angle between vectors $u$ and $v$ is

Example 1. Let $u = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $v = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$. Find the angle $\theta$ between these vectors.
The inner product, or dot product of vectors \( u \) and \( v \) on \( \mathbb{R}^2 \) (or \( \mathbb{R}^3 \)) are defined by

Remark

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Definition  Two vectors \( u \) and \( v \) in \( \mathbb{R}^2 \) or \( \mathbb{R}^3 \) are called

Example 2. The vectors \( u = \begin{bmatrix} 2 \\ -4 \end{bmatrix} \) and \( v = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \) are orthogonal.

Let \( u \), \( v \) and \( w \) be vectors in \( \mathbb{R}^2 \) or \( \mathbb{R}^3 \), and \( c \) be a scalar. The inner product satisfies:

1. 
2. 
3. 
4. 
Unit Vector

A vector in $\mathbb{R}^2$ or $\mathbb{R}^3$ whose length is 1 is called

If $\mathbf{v}$ is any nonzero vector, then a unit vector in the direction of $\mathbf{v}$ is

Example 3. Let $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$. Find a unit vector in $\mathbb{R}^3$ which

1. is in the same direction as $\mathbf{v}$
2. is in the opposite direction as $\mathbf{v}$
3. is orthogonal to $\mathbf{v}$
4. has an angle of 60° between $\mathbf{v}$. 