6. Prove that \(1 \cdot 1! + 2 \cdot 2! + \cdots + n \cdot n! = (n + 1)! - 1\) whenever \(n\) is a positive integer.

8. Prove that \(2 - 2 \cdot 7 + 2 \cdot 7^2 - \cdots + 2(-7)^n = (1 - (-7)^{n+1})/4\) whenever \(n\) is a nonnegative integer.

24. Prove that \(1/(2n) \leq [1 \cdot 3 \cdot 5 \cdots (2n - 1)]/(2 \cdot 4 \cdots 2n)\) whenever \(n\) is a positive integer.

51. What is wrong with this "proof"?
   "Theorem" For every positive integer \(n\), if \(x\) and \(y\) are positive integers with \(\text{max}(x, y) = n\), then \(x = y\).

   Basis Step: Suppose that \(n = 1\). If \(\text{max}(x, y) = 1\) and \(x\) and \(y\) are positive integers, we have \(x = 1\) and \(y = 1\).

   Inductive Step: Let \(k\) be a positive integer. Assume that whenever \(\text{max}(x, y) = k\) and \(x\) and \(y\) are positive integers, then \(x = y\). Now let \(\text{max}(x, y) = k + 1\), where \(x\) and \(y\) are positive integers. Then \(\text{max}(x - 1, y - 1) = k\), so by the inductive hypothesis, \(x - 1 = y - 1\). It follows that \(x = y\), completing the inductive step.

54. Use mathematical induction to show that given a set of \(n + 1\) positive integers, none exceeding \(2n\), there is at least one integer in this set that divides another integer in the set.