MA 301 Practice Test 4, Spring 2004

You should bring a calculator to the test to be able to do problems such as Problem 5

(1) State the “official” definition of “lim_{x→a} f(x) = L.”

(2) State (carefully) the Sum Theorem for limits of functions from Chapter 10. Then prove it.

(3) Suppose that f(x) and g(x) are both continuous at x = a. Prove that h(x) = f(x)g(x) is also continuous at x = a. You may use the product theorem for limits of functions from Chapter 10.

(4) Prove that Z = 1/(3π + 5) is irrational. You may assume that π is irrational. You MAY NOT use Proposition 1 from Chapter 9.

(5) Find

(a) an explicit irrational number Z satisfying 17/13 < Z < 18/13. You need not prove that Z is irrational.

(b) an explicit rational number Z satisfying π < Z < 22/7.

(6) Find an explicit one-to-one correspondence between the sets A and B where:

(a) A = (−1, 3) and B = (0, 1).

(b) A is the set of even natural numbers and B is the set of odd natural numbers.

(c) A is the set of natural numbers which are multiples of 2 and B is the set of natural numbers which are multiples of 3.

(7) Use a δ-ε argument to prove the following limit statements:

(a) \( \lim_{x→2} \frac{3x}{x + 1} = 2 \)

(b) \( \lim_{x→2} \frac{1}{x^2} = \frac{1}{4} \)

(c) \( \lim_{x→1} (x^2 + 3) = 4 \)

(d) \( \lim_{x→1} \frac{1}{x^2 + 3} = \frac{1}{4} \)

(e) \( \lim_{x→0} \frac{1}{1 - x} = 1 \)

(f) \( \lim_{x→1} \frac{1}{\sqrt{3 + x}} = \frac{1}{2} \)
(8) Assume that \( \lim_{x \to a} f(x) = 2 \). Use a \( \delta-\epsilon \) argument to prove:

(a) \( \lim_{x \to a} \frac{2}{f(x) + 2} = \frac{1}{2} \)
(b) \( \lim_{x \to a} \sqrt{f(x)} + 2 = 2 \)
(c) \( \lim_{x \to a} f(x)^2 = 4 \)
(d) \( \lim_{x \to a} \frac{3f(x)}{f(x) + 1} = 2 \)

(9) Find a value of \( a \) for which the following function is continuous at \( x = 1 \).

\[
f(x) = \begin{cases} 
  x + a & x < 2 \\
  x^2 & x \geq 2 
\end{cases}
\]