1. (a) (4pts) Let $A = [a_{ij}]_{1 \leq i,j \leq 4}$ be a $4 \times 4$ matrix. Write down the full expression of $\det(A)$ in terms of the $a_{ij}$’s using the definition of the determinant function. (Be careful about the signs in front of different terms).

(b) (6pts) Let

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -2 & 1 & 2 \\ 1 & 4 & 1 & 4 \\ -1 & -8 & 1 & 8 \end{bmatrix}.$$ 

Compute using row reductions the determinant of $A$. 

Write your name in the top left corner. Attempt all questions. You must show all work in order to obtain credit.
2. (10pts) Let $E$ be an $n \times n$, $n > 1$, matrix whose $(1, n)$-th entry is 1 and the rest of whose entries are all 0. For each of the following matrices either justify in one sentence that it is not invertible, or write down its inverse.

(a) $E$:

(b) $E^2$:

(c) $I_n + E$:

(d) $I_n - E$:

(e) $(I_n + E)^3$: 
3. (10 pts) Consider the system of linear equations given by $Ax = b$ where

$$A = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 \\
1 & 0 & 1 & 2 & 3 \\
1 & 0 & 1 & 4 & 9
\end{bmatrix},$$

and

$$b = \begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix}.$$

Describe the solutions of the system.
4. Let $A$ be an $n \times n$ matrix such that $A^3 = 0$.

(a) (3pts) Is $A$ invertible? Justify.

(b) (3pts) Is $A + A^2$ invertible? Justify.

(c) (4pts) Is $I_n - A$ invertible? Justify.
5. Let $P_3$ be the set of polynomials in $X$ of degree at most 3 having real coefficients.

(a) (2 pts) Is $P_3$ a vector space (where the operation $\oplus$ is the usual polynomial addition and the operation $\odot$ is the usual multiplication of polynomials by real numbers)? Justify.

(b) (2pts) Let $V = \{ f \in P_3 \mid f(0) = 1 \}$. Is $V$ a subspace of $P_3$? Justify.

(c) (2pts) Let $W = \{ f \in P_3 \mid f(0) = f(1) \}$. Is $W$ a subspace of $P_3$? Justify.

(d) (2pts) Let $S = \{ f \in P_3 \mid f(0) = 2f(1) + 1 \}$. Is $S$ a subspace of $P_3$? Justify.

(e) (2pts) Let $T = \{ f \in P_3 \mid f'(0) = 2f(1) \}$ (where $f'$ is the derivative of $X$ with respect to $X$). Is $T$ a subspace of $P_3$? Justify.