Final Exam – Part 2 (Due April 29)

This is part of the take-home final exam. All students are expected to complete the project independently.

1. Consider the following autonomous system

\[
\frac{du}{dt} = f(u) + g(u)
\]

and the linear multistep IMEX scheme:

\[
u^{n+1} + \sum_{j=0}^{k-1} \alpha_j u^{n-j} = \tau \sum_{j=0}^{k-1} \beta_j f(u^{n-j}) + \tau \sum_{j=-1}^{k-1} \gamma_j g(u^{n-j}),
\]

where \(\gamma_{-1} \neq 0\).

Find the necessary conditions so that the scheme is of \(p\)-th order accurate for \(p \leq k\).

2. Give a summary (not exceeding two pages) about the main properties (stability, advantage, dis-advantage, etc.) of multistep schemes, Runge-Kutta schemes, exponential splitting schemes and IMEX schemes.