SEC 6.1

9. Let $g(x_i) = f(x_i)$ for $0 \leq i \leq n - 1$ and $h(x_i) = f(x_i)$ for $1 \leq i \leq n$. Set $k(x) = g(x) + ((x_0 - x)/(x_n - x_0))[g(x) - h(x)]$. Then $k(x_0) = g(x_0) = f(x_0)$ and for $1 \leq i \leq n - 1$ we have $k(x_i) = g(x_i) + ((x_0 - x_i)/(x_n - x_0))[g(x_i) - h(x_i)] = g(x_i) = f(x_i)$ and $k(x_n) = h(x_n) = f(x_n)$.

14. Say $x_j = 0$. $|p(x) - f(x)| = \left| (1/n!) f^{(n)}(\xi) \prod_{i=0}^{n-1} (x - x_i) \right|$

$\leq (1.5431/n!)|x| \prod_{i=0}^{n-1} (x - x_i) \leq (1.5431/n!)|x|2^{n-1}$ since node $x_j$ is 0. Note as in Problem 6.1.13,

$f^{(n)}(\sinh x) = \begin{cases} \sinh x & \text{n even} \\ \cosh x & \text{n odd} \end{cases}$

and $|f^{(n)}(\sinh x)| \leq \max\{|\sinh 1, \cosh 1\}$, on $[-1, 1]$.

So $|p(x) - f(x)|/|f(x)| \leq (1.5431/n!)|x/\sinh x|2^{2n-1} \leq (1.5431/n!)2^{n-1} \leq (2^n/n!)$

since $x/\sinh x \leq 1$.

22. Lagrange form: $p(x) = -(1/2)(x + 2)(x - 1) - (1/3)x(x + 2) = -(1/6)(5x^2 + 7x - 6)$.

Newton form:

\[
\begin{array}{cccc}
  x & f(x) \\
-2 & 0 & 1/2 & -5/6 \\
0 & 1 & -2 \\
1 & -1 \\
\end{array}
\]

$p(x) = (1/2)(x + 2) - (5/6)(x + 2)x = -(1/6)(5x^2 + 7x - 6)$.

27. $|f(x) - p(x)| \leq |f^{(13)}(\xi)/(13!)| \prod_{i=0}^{12} |x - x_i|$. Now $f(x) = f^{(13)}(x) = e^{x-1}$ and $|f(\xi)| \leq f(1) = e^0 = 1$

for $\xi \in [-1, 1]$. Also, $\prod_{i=0}^{12} |x - x_i| \leq 2^{13}$. Therefore, $|f(x) - p(x)| \leq 2^{13}/(13!) = 1.315 \times 10^{-6}$. 
SEC 6.2

5. The unique polynomial of degree at most \( n \) that interpolates \( p \) at \( x_0, x_1, \ldots, x_n \) is \( p \) itself. Hence, the desired equation is Equation (10), with \( f = p \).

6. (Proof by induction). For \( n = 1 \), \((\alpha f + \beta g)[x_0, x_1] = ((\alpha f + \beta g)(x_1) - (\alpha f + \beta g)(x_0))/(x_1 - x_0)
\]
to \((\alpha f + \beta g)(x_1) - (\alpha f + \beta g)(x_0))/(x_1 - x_0) = \alpha f[x_0, x_1] + \beta g[x_0, x_1].\) Suppose it is true for \(2, 3, \ldots, n\). Consider, \((\alpha f + \beta g)[x_0, x_1, \ldots, x_{n+1}]
\]
to \((\alpha f + \beta g)[x_1, \ldots, x_{n+1}] - (\alpha f + \beta g)[x_0, \ldots, x_n])/(x_{n+1} - x_0)
\]
and \(\alpha f[x_1, \ldots, x_{n+1}] - \alpha f[x_0, \ldots, x_n] + \beta g[x_1, \ldots, x_{n+1}] - \beta g[x_0, \ldots, x_n])/(x_{n+1} - x_0)
\]
= \(\alpha f[x_0, \ldots, x_{n+1}] + \beta g[x_0, \ldots, x_{n+1}].\)

17.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3/2</td>
</tr>
<tr>
<td>3/2</td>
<td>13/4</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>5/3</td>
</tr>
</tbody>
</table>

Thus, \( p(x) = 3 + (1/2)(x - 1) + (1/3)(x - 1)(x - 3/2) - 2(x - 1)(x - 3/2)(x). \)

19. At \( x_0 \), we have \( [(x_n - x_0)u(x_0) + (x_0 - x_0)v(x_0)]/(x_n - x_0) = u(x_0) = f(x_0). \) For \(1 \leq i \leq n - 1\), we have \( [(x_n - x_i)u(x_i) + (x_i - x_0)v(x_0)]/(x_n - x_0) = (x_n - x_i)f(x_i)/(x_n - x_0) = f(x_i). \) At \( x_n \), we have \( [(x_n - x_n)u(x_n) + (x_n - x_0)v(x_0)]/(x_n - x_0) = v(x_n) = f(x_n). \)