MATH 692: Project 2
Due date: Sep. 25

1. Spectral-collocation:

Let \( \{x_i\}_{i=0}^{N} \) be the set of Chebyshev-Gauss-Lobatto points. Use the subroutine col_bvp2.m in the subdirectory “general” to solve Problem 1 on Page 98.

2. Multi-domain collocation method:

(i) Let \( a = x_0 < x_1 < \cdots < x_K = b \), and denote \( I_j = (x_{j-1}, x_j) \) for \( j = 1, 2, \cdots, K \).

Write a program for the multi-domain collocation method using \((N_j + 1)\) points on each subinterval \( I_j \) for the problem:

\[-u_{xx} + p(x)u_x + q(x)u = f, \quad x \in (a, b);\]
\[a_- u(a) + b_- u'(a) = c_-, \quad a_+ u(b) + b_+ u'(b) = c_+.\]

Test your program with a simple exact solution.

(ii) Take \( x_0 = a = -1, x_1 = -0.2, x_2 = 0, x_3 = 0.2, x_4 = b = 1 \). Use your program to solve the problem

\[-u_{xx} + u(x) = f(x), \quad u(-1) = u(1) = 0\]

with the exact solution \( u(x) = \exp(-100x^2) \) (you need to compute the corresponding \( f(x) \)). Determine a “good” combination of \( \{N_j\}_{j=1}^{4} \) such that the maximum error at these collocation points is less than \( 10^{-6} \). Compare your result with the one-domain approach.

1. Preconditioned spectral-collocation:

(i) Write a finite-difference code for solving the equation (2.4.1)-(2.4.2) on the Chebyshev-Gauss-Lobatto points.

(ii) Compute the condition numbers of the preconditioned matrix \( A_{fd}^{-1} A_{sp} \) for \( N = 2^k \) with \( k = 3, 4, 5, 6, 7, 8 \), where \( A_{fd} \) (resp. \( A_{sp} \)) is the matrix associated with the finite difference (resp. spectral-collocation) method.

(iii) (optional) Use the preconditioned BICG iterative method (in Matlab: bicc with a function handle) to build a program for solving (2.4.1)-(2.4.2) in \( O(N \log N) \) operations. Redo the Problem 1 in Page 98 by this fast program.