Math 265, Practice Midterm 2

Name: _________________________________________
Section: _________

This exam consists of 8 pages including this front page.

Ground Rules
1. No calculator is allowed.
2. Show your work for every problem unless otherwise stated.
3. You may use one 4-by-6 index card, both sides.

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1. The following are true/false questions. You don’t have to justify your answers. Just write down either T or F in the table below. $A$, $X$, $b$ are always matrices here.

(a) It is not possible to find 4 linearly independent vectors $v_1, v_2, v_3, v_4$ in $\mathbb{R}^3$.
(b) Let $A$ be an $m \times n$ matrix. If the dimension of the column space of $A$ is 5, then $n \geq 5$.
(c) Consider a system of linear equations $AX = b$ with $A$ an $n \times n$-matrix. If the system is consistent then $\text{rank}(A) = n$.
(d) Let $V$ be an inner product space and $u, v, w \in V$. If $w$ is orthogonal to $u$ and $v$, then $w$ is orthogonal to any linear combination of $u$ and $v$.
(e) Let $A$ be an $n \times n$-matrix. If $\text{rank}(A) < n$ then $\det(A) = 0$.

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2. Quick Questions, no steps of explanation are needed.

(a) Suppose \( u = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \) and \( v = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \). Find the length of \( u \) and \( v \).

(b) Let \( u \) and \( v \) be as the above question and \( \theta \) is the angle between \( u \) and \( v \). Find \( \cos(\theta) \).

(c) Find a basis for the space of \( 3 \times 3 \) real symmetric matrices.
3. (a) For what values of $c$ are the vectors \[
\begin{pmatrix}
-1 \\
0 \\
-1
\end{pmatrix}, \begin{pmatrix}
2 \\
1 \\
2
\end{pmatrix} \text{ and } \begin{pmatrix}
1 \\
1 \\
c
\end{pmatrix}
\] in $\mathbb{R}^3$ linearly independent?

(b) If possible, find $a, b, c$ so that $u = \begin{pmatrix}
a \\
b \\
c
\end{pmatrix}$ is orthogonal to $v = \begin{pmatrix}
1 \\
2 \\
1
\end{pmatrix}$ and $w = \begin{pmatrix}
1 \\
-1 \\
1
\end{pmatrix}$. 
4. Let $A = \begin{pmatrix} 1 & 0 & -1 & 1 \\ 1 & -1 & 0 & -1 \\ 2 & -1 & -1 & 0 \end{pmatrix}$.

1. Find a basis for the row space of $A$.

2. Find a basis for the column space of $A$.

3. Find a basis for the null space of $A$.

4. Verify the equality $\text{rank}(A) + \text{Nullity}(A) = n$. 
5. Let $V \subset \mathbb{R}^4$ be a subspace spanned by \[
\begin{bmatrix}
1 & 1 & 0 & 1 \\
1 & 0 & 1 & 0 \\
0 & 1 & 1 & 0 \\
1 & 0 & 0 & 1
\end{bmatrix}.
\]

1. Find an orthonormal basis of $V$.

2. Let $u = \begin{pmatrix} 0 & -1 & 1 & 0 \end{pmatrix}^T$. Find $\text{Proj}_V u$. 
6. Consider the following system of linear equation

\[-2x_1 + 3x_2 - x_3 = 1\]
\[x_1 + 2x_2 - x_3 = 4\]
\[-2x_1 - x_2 + x_3 = -3\]

1. Find the rank of the coefficient matrix.

2. Find the rank of the augmented matrix.

3. Is the system consistent? why?
7. Let $A$ be a $3 \times 5$-matrix.

1. What will be the maximal possible rank of $A$.

2. Show that columns of $A$ are linearly dependent.

3. What will be the maximal possible dimension of $N(A)$.