1. Recall the countably-additivity property of a probability measure $P$: $P(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i)$ for any countable collection of mutually exclusive events $A_i$. Prove that the above property is equivalent to the following two statements:

   (a) (continuity from below) for any countable collection of “increasing” events: $B_1 \subseteq B_2 \subseteq B_3 \subseteq \cdots$, it holds that
   
   $$P\left(\bigcup_{i=1}^{\infty} B_i\right) = \lim_{n \to \infty} P(B_i)$$

   (b) (continuity from above) for any countable collection of “decreasing” events: $C_1 \supseteq C_2 \supseteq C_3 \supseteq \cdots$, it holds that
   
   $$P\left(\bigcap_{i=1}^{\infty} C_i\right) = \lim_{n \to \infty} P(C_i)$$

2. Let $x_1x_2x_3\cdots$ be an infinite sequence of strings of 0 or 1’s. Let $S_n = x_1 + x_2 + \cdots + x_n$, i.e. $S_n$ equals to the number of 1’s in the first $n$ positions. Give an example of a string such that the limit $\lim_{n \to \infty} \frac{S_n}{n}$ does not exist.