1. (20pts) Given a sphere of radius 2 centered at the origin, find the equation for the plane that is tangent to it at the point \((1, 1, \sqrt{2})\) by considering the sphere as:

(a) a surface parametrized by \(\Phi(\theta, \phi) = (2 \cos \theta \sin \phi, 2 \sin \theta \sin \phi, 2 \cos \phi)\);

(b) a level surface of \(f(x, y, z) = x^2 + y^2 + z^2\); and

(c) the graph of \(g(x, y) = \sqrt{4 - x^2 - y^2}\).
2. (20pts) The cylinder $x^2 + y^2 = \frac{1}{4}$ divides the unit sphere $S$ into two regions $S_1$ and $S_2$, where $S_1$ is inside the cylinder and $S_2$ is outside. Find the ratio of areas $A(S_2)/A(S_1)$.

3. (15pts) Evaluate $\int \int_S z \, dS$, where $S$ is the surface $z = (x^2 + y^2)/2$ with $x^2 + y^2 \leq 1$. 

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4. (20pts) Let $S$ be the part of the cone $z^2 = x^2 + y^2$ with $z$ between 1 and 2 oriented by the normal pointing out of the cone. Compute $\int \int_{S} \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x, y, z) = (x, y, 2z)$.

5. (15pts) Evaluate $\int_{C^+} x^3 \, dy - y^3 \, dx$ where $C^+$ is the unit circle ($x^2 + y^2 = 1$) in the counterclockwise direction.
6. (15pts) Evaluate \( \int \int_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S} \) where \( \mathbf{F} = (x + z^2, x + y + z, z + x) \) and \( S \) is the surface \( z = x^2 + y^2 - 9 \) with \( z \leq 0 \) and \( S \) is oriented by the outward (= downward) unit normal.

7. (15pts) Evaluate the surface integral \( \int \int_S \mathbf{F} \cdot d\mathbf{S} \) where \( \mathbf{F} = x^2\mathbf{i} + (\log z + y)\mathbf{j} + (e^x + z)\mathbf{k} \) and \( S \) is the surface of the unit cube in the first octant (i.e., \( 0 \leq x \leq 1 \), \( 0 \leq y \leq 1 \), and \( 0 \leq z \leq 1 \)) with the outward orientation.
8. (20pts) (a) Compute the curl of \( \mathbf{F} = (y, x + z, y + \cos z) \).
(b) Find a scalar function \( f(x, y, z) \) such that \( \nabla f = \mathbf{F} \).
(c) Compute \( \int_{c} y \, dx + (x + z) \, dy + (y + \cos z) \, dz \) along the path \( c(t) : x = \cos^3 t, y = \sin^3 t, z = t^3, \ 0 \leq t \leq \frac{\pi}{2} \).

9. (10pts) Let \( D \) be a \( y \)-simple region and let \( C \) be its boundary. Suppose \( P : D \to R \) is of class \( C^1 \) and \( Q = 0 \). Prove that \( \int_{C} P \, dx + Q \, dy = -\int_{D} \frac{\partial P}{\partial y} \, dxdy \).