Introduction to Partial Differential Equations
Instructor: Professor Patricia Bauman
Course Number: MA 52300
Credits: Three
Time: 12:30–1:20 PM MWF

Description

Development of qualitative properties for solutions to the Laplace, the wave and the heat equations and methods for representing solutions. First-order quasi-linear and nonlinear equations and their applications. The Cauchy-Kovalevsky theorem. Characteristics, classification and canonical forms of linear equations. Equations of mathematical physics.

Textbook: Partial Differential Equations: 2nd Edition (Graduate Studies in Mathematics) by Lawrence C. Evans

Probability Theory II
Instructor: Professor Samy Tindel
Course Number: MA 53900
Credits: Three
Time: 4:30–5:45 PM TTh

Description

This course introduces various crucial notions concerning discrete and continuous time random functions. It has to be seen as a continuation of MA 538. It can be roughly divided in 3 parts.

(1) We begin by a quick review of conditional expectation. Then we will introduce the notion of martingale, which is a class of stochastic processes arising in the description of fair games. We will study the convergence properties of this kind of object, when the time index is discrete.

(2) The next topic to be covered is a construction and analysis of the most important continuous time stochastic process, namely Brownian motion. We shall also introduce It's integral with respect to a Brownian motion and the basis of stochastic calculus.

(3)) The last part of the course concerns ergodic theorems, which can be seen a very general tool allowing to get limit theorems for sequences of random variables.

As we did for MA 538, we will mostly follow Durrett's book: Probability – Theory and Examples. The stochastic calculus part will be based on Karatzas and Shreve's book: Brownian motion and stochastic calculus.

Distributions, Fourier Transform and Pseudo–Differential Operators

Instructor: Professor Plamen Stefanov

Course Number: MA 59800APS

Credits: Three

Time: 1:30–2:45 PM TTh

Description

This course will start with an introduction to distributions and the Fourier transform. We will define distributions, study their major properties, convolutions, pull–backs and push-forwards. We will spend some time on homogeneous distributions, their Fourier transforms, Riesz potentials and applications to integral geometry. We will also discuss the Schwartz kernel theorem and fundamental solutions of some basic PDEs. Distributions will be presented form the point of view of their fundamental role in analysis rather than as a curious topological space (but the topology will not be ignored). We will introduce Sobolev spaces as well.

The second part of the course will be an introduction to pseudo–differential operators. One motivation to study them is that they appear naturally as inverses of elliptic differential operators. The appear naturally in integral geometry as well. We will introduce the basics of the calculus, the construction of a parametrix of elliptic differential operators, and some applications to the Radon transform if there is time left for that.

Prerequisites are real analysis and some background in functional analysis. Several texts will be used (Friedlander–Joshi, Hormander, Taylor, Helgason) and instructor's notes for at least some of the topics covered. The course is aimed at beginning students with interest in analysis and applied math but of course, all students are welcome.

Introduction to the Representation Theory of Finite Groups

Instructor: Professor Kenji Matsuki

Course Number: MA 59800ARFG

Credits: Three

Time: 11:30 AM–12:20 PM MWF

Description

The purpose of this course is to give a concise introduction to the basics of the representation theory of finite groups. The represenation theory of groups, not just finite groups but various profinite groups (e.g., the absolute Galois group), algebraic groups, topological groups, Lie groups, etc., occupies such a central role in modern mathematics. For example, it is probably impossible to talk about the recent developments in number theory without referring to the representation theory of groups. The representation theory of finite groups is the most accessible entrance to this vast and beautiful subject, while being as deep as the theory of finite groups itself. However, an introductory course on this subject tends to be missing from the standard curriculum of a graduate program, creating a hole (at least here at Purdue University, it seems). The reason maybe, I imagine, it is too easy for the hardcore representation theorists to teach, and at the same time it might be too scary for the outsiders to discuss this deep subject without having a global view only attainable through expertise. If one allows me to exaggerate the goal of this course, then it would be to fill this hole. But at the more practical and modest level, the goal is to cover the basics of basics from an amateur point of view (I'm an amateur of Representation Theory!), using the two "bible–like" textbooks in the subject at a leisurely pace, in a relaxed and friendly atmosphere with the minimum background required, learning together with the students. The only catch is to add a "training–camp–like" flavor, where the emphasis is on "getting one's hand dirty" by working on the selected exercise problems.

(i) Textbooks:

- **Linear Representation of Finite Groups** by J.P. Serre, GTM 42, Springer
- **Representation Theory** by William Fulton and Joe Harris, GTM 129, Springer

(ii) Classes:

- Lectures on Mondays, Wednesdays, and Fridays 11:30 - 12:20 in REC 313
- Once every month a couple of classes will be dedicated to the presentation of the solutions to some of the exercise problems by STUDENTS.

(iii) Grading scheme:

- I will assign 2–4 problems per week as homework, and the students are supposed to turn in their answers to be graded.
- In order to get A, a student has to give at least one presentation during the semester (or submit a report at the end of the semester).

Topological Data Analysis
Instructor: Professor Ralph Kaufmann
Course Number: MA 59800ATDA
Credits: Three
Time: 9:00–10:15 AM TTh

Description

This will be an introduction to topological data analysis. The goal is to provide an understanding of the basic concepts and further more advanced topics. The basic concepts will include persistent homology and bar codes as well as the underlying topological notions. Further topics may include visualisation using the Mapper algorithm, principal component analysis, and various clustering algorithms.

References will be various books, articles and lecture notes which will presented on the course web page. These will include: Computational Topology by Herbert Edelsbrunner and John L. Harer, "Topological pattern recognition for point cloud data". Acta Numerica. 23: 289–368 by Carlsson, Gunnar.

Methods of Linear and Nonlinear Partial Differential Equations I

Instructor: Professor Harold Donnelly

Course Number: MA 64200

Credits: Three

Time: 1:30–2:20 PM MWF

Description

Second order elliptic equations, maximum and comparison principles, Dirichlet problem, interior and boundary regularity, Schauder and Sobolev estimates, weak solutions, Harnack inequality, Fredholm alternative.

Textbook – Elliptic partial differential equations of second order by Gilbarg and Trudinger, second edition, Springer.

Introduction to Algebraic Geometry

Instructor: Professor Jaroslaw Wlodarczyk

Course Number: MA 66500

Credits: Three

Time: 4:30–5:45 PM TTh

Description

We shall give a basic course in Algebraic Geometry based upon Mumford's Red book, and Hartshorne'es Algebraic Geometry: Chapters 2 and part of Chapters 3. It is meant as the expansion and continuation of the spring 2018 course given by Prof. Arapura., though there will be several overlaps. Since the main concepts will be reviewed, the new students are welcome. The important part of the class will be solving problems from Hartshorne's book and others.

The tentative list of the topics includes but is not limited to:

- Schemes and Varieties.
- Morphisms of schemes: proper, separated and finite.
- Nonsingularity , UFD property, tangent space, tangent cone, differentials.
- Etale morphisms.
- Normal varieties, Normalization
- Zariski main Theorem
- Flat and smooth morphisms
- Coherent and Quaicoherent modules
- Cohomology of Sheaves
- Divisors

Homework: Problem solving is vital for this class. I will actively seek groups of volunteers to report their solutions in the problem sessions. The hwk problems will be assigned on Thursday and will be due on the next Thursday.

There will be usually a 30-45 minutes problems session held during Thursday lecture. During this time the (previously selected) volunteers are going to present their solutions.

Regarding Commutative Algebra: It is very useful to have basic working knowledge of commutative algebra, at the level of Introduction to Commutative Algebra by Atiyah and Macdonald, before plunging into Hartshorne's book.

Exams: No exam.

Introduction to Lie Groups, Lie Algebras and Their Representations
Instructor: Professor Saugata Basu
Course Number: MA 69000B
Credits: Three
Time: 12:00–1:15 PM TTh

Description

I will cover the basic theory of Lie groups and algebras, including solvability, semi–simplicity, and the Cartan–Killing classification of semi–simple Lie algebras via their Dynkin diagrams.

We will then study the representation theory of Lie groups and algebras, weight spaces, highest weight theory, Schur–Weyl duality, Weyl character formula, Verma modules and BGG resolutions. If time permits I will discuss some applications of the classical theory in more modern applied areas – such as tensor decompositions and geometric complexity theory.

References:

1. Lie Groups Beyond an Introduction – A. W. Knapp.
2. Representation theory – A first course, Fulton and Harris.
3. An introduction to Lie Groups and Lie Algebras, A. Kirillov, Jr.

Finite Groups
Instructor: Professor Louis de Branges
Course Number: MA 69000D
Credits: Three
Time: 9:30–10:20 AM MWF

Description

I would like graduate students for a doctoral program on molecular groups. These are finite groups which describe the symmetries of the electron configuration of a chemical molecule. They satisfy a hypothesis relating to the quaternion group of eight elements plus or minus i, j, k, and 1.The Fourier analysis on the locally compact Abelian groups constructed from the molecule contains a general class of zeta functions for which a proof of the Riemann hypothesis is given in a still incomplete manuscript. I expect funding by pharmaceutical companies who would apply the theory for the design of new medications.. I am still in the preliminary stages of the theory but believe that it is time to begin a graduate program.

Students need to have completed qualifying examinations and have an interest in Fourier analysis and in finite groups.

I will teach an introductory course MWF at nine-thirty in the fall semester if there is sufficient registration.

Groebner Bases in Comm Algebra
Instructor: Professor Giulio Caviglia
Course Number: MA 69000GBCA
Credits: Three
Time: 12:30–1:20 PM MWF

Description

Topics in Spectral Methods and Computational Fluid Dynamics
Instructor: Professor Jie Shen
Course Number: MA 69200A
Credits: Three
Time: 9:00–10:15 AM TTh

Description

The course consists of two parts:

Part 1: Introduction to spectral methods

1. Orthogonal polynomial and related approximation theory
2. Spectral methods for elliptic equations
3. Spectral methods for fractional differential equations

Part 2: Efficient algorithms for incompressible flows, phase–field models and related problems

1. Incompressible Stokes and Navier–Stokes equations: basic tools and results
2. Projection type methods for Navier–Stokes equations
3. Gradient flows
4. Phase–field models for multiphase complex fluids

Quantitative Geometric and Functional Inequalities

Instructor: Professor Emanuel Indrei

Course Number: MA 69300A

Credits: Three

Time: 1:30–2:45 PM TTh

Description

In recent years it has been of interest to study functional or geometric inequalities which quantify proximity to optimizers. For instance, the isoperimetric inequality states that given a set $E \subset \mathbb{R}^n$ such that $|E| = |B_1|$,

$$P(E) \geq P(B_1),$$

where B_1 is the unit ball in $\mathbb{R}^n$, $P(E)$ denotes the perimeter of E, and $|E|$ denotes the Lebesgue measure of E; equality holds if and only if E is a ball. Such a result is classical and there has been an effort to understand sharp perturbative results: if $\delta(E) := \frac{P(E)}{P(B_1)} - 1$, then does $\delta(E) << 1$ imply that E is close to B_1 (up to a translation)? The difficulty of such questions involve understanding the appropriate conditions for which

(i) the inequality makes sense, i.e. the necessary regularity assumptions to define the perlimeter;

(ii) how to quanity proximity to a ball;

(iii) whether the quantification in (ii) is optimal.

These issues have been classical problems in the calculus of variations and it turns out that not all quantifications make sense, for instance if one considers the Hausdorff metric, then a ball with a thin long neck attached to it will produce a large distance to an optimizer although $\delta(E)$ could be made arbitrarily small. Therefore, finding an appropriate metric is part of the problem: given two sets A and B, let

$$A \Delta B = (A \backslash B) \cup (B \backslash A)$$

denote the symmetric difference. The following inequality holds for sets of finite perimeter that satisfy $|E| = |B_1|$:

$$\inf_{x \in \mathbb{R}^n} |E \Delta (x + B_1)|^2 \leq c_n \delta(E),$$

where the exponent 2 is sharp in every dimension and $c_n > 0$ is a dimensional constant (this settled Hall's conjecture) and it is surprising that the sharp exponent is dimension independent. There are several proofs which involve symmetrization techniques, optimal transport theory, and partial differential equations and we will study some of these methods and also several extensions to anisotropic settings in which the minimizer is a convex set instead of a ball. We will also consider analogous quantitative results for the Sobolev, logarithmic Sobolev, and Brunn–Minkowski inequalities.

Course grade

The course grade is based on an in–class presentation.

Mixed and Least–Squares Finite Element Methods for Systems of Partial Differential Equations

Instructor: Professor Zhiqiang Cai

Course Number: MA 69400A

Credits: Three

Time: 10:30–11:45 AM TTH

Description

The original physical equations for mechanics of continua are systems of partial differential equations of first–order. There are many advantages to simulate these first-order systems directly. This can be done through either mixed or least–squares finite element methods. This course is an introduction to both techniques, with applications to Darcy's flow in porous media, elastic equations for solids, incompressible Newtonian fluid flow, and Maxwell's equations in electromagnetic. We shall focus on fundamental issues such as (mixed and least–squares) variational formulations, construction of finite element spaces of H(div) and H(curl), and a posteriori error estimators for self–adaptive finite element methods.

A tentative list of contents:

1. Adaptive Finite Element Method
2. Mathematical Models of Continuum Mechanics
3. Construction of Finite Element Spaces in H(div) and H(curl)
4. Mixed Variational Formulations
5. Least–Squares Variational Formulations
6. Finite Element Approximations

Prerequisite: MA/CS 615 or equivalent or consent of instructor.

[1] F. Brezzi and M. Fortin, Mixed and Hybrid Finite Element Methods, Springer–Verlag, New York, 1991.

[2] V. Girault and P. Raviart, Finite Element Methods for Navier–Stokes Equations: Theory and Algorithms, Springer–Verlag, New York, 1986.

[3] P. Monk, Finite Element Methods for Maxwell?s Equations, Oxford University Press, Oxford, 2003.

[4] R. Verfurth, A posteriori error estimation techniques for finite element methods, Numerical Mathematics and Scientific Computation, Oxford University Press, Oxford, 2013.

[5] research articles and my lecture notes.