Ordinary Differential Equations and Dynamical Systems

Instructor: Professor Nung Kwan Yip
Course Number: MA 54300
Credits: Three
Time: 9:00–10:15 AM TTh

Description

This is a beginning graduate level course on ordinary differential equations. It covers basic results for linear systems, local theory for nonlinear systems (existence and uniqueness, dependence on parameters, flows and linearization, stable manifold theorem) and their global theory (global existence, limit sets and periodic orbits, Poincare maps). Some further topics include bifurcations, averaging techniques and applications to Hamiltonian mechanics and population dynamics.

Prerequisites: one undergraduate course in each of the following topics: linear algebra (for example, MA 265, 351), differential equation (for example, MA 266, 366), analysis (for example, MA 341, 440, 504), or instructor's consent.

Introduction to Functional Analysis

Instructor: Professor Marius Dadarlat
Course Number: MA 54600
Credits: Three
Time: 12:30–1:20 PM MWF

Description

1. Banach spaces
2. Hilbert spaces
3. Linear Operators and functionals
4. The Hahn-Banach Theorem
5. Duality
6. The Open Mapping Theorem
7. The Uniform Boundedness Principle
8. Weak Topologies
9. Spectra of operators
10. Compact operators
11. Banach algebras and C*-algebras
12. Riesz calculus
13. Fredholm index

14. Gelfand transform
15. Spectral theorem for normal operators

If time allows:

16. Unbounded Operators
17. Applications: Differential operators, Peter-Weyl theorem

Prerequisites: Familiarity with basic measure theory

Grading:

Attendance 35%,

HW 40%,

Final Exam 25% (a take-home 36 hours no collaboration exam).

No specific textbook is required. These topics are covered by most books on functional analysis. A good reference is: Gert Pedersen, *Analysis Now*, (Graduate Texts in Mathematics) *Corrected Edition*!

Homological Algebra
Instructor: Professor Donu Arapura
Course Number: MA 55800
Credits: Three
Time: 12:00–1:15 AM TTh

Description

The official designation for this course is "Abstract Algebra II", which permits a certain amount of latitude. So I've decided to cover homological algebra. This is the part of algebra that came out of algebraic topology in the 1940's and 50's. Its impact cannot be underestimated. Not only is it important in algebra, but within a few years, the foundations of subjects such as class field theory and algebraic geometry were essentially rewritten to use this viewpoint.

For prerequisites, I won't assume much more than a good knowledge of basic algebra (if you've been through Atiyah-Macdonald for example, that should be plenty). In particular, since I won't assume prior knowledge of algebraic topology, I will cover a bit of that for purposes of motivation. Then I will alternate between pure homological algebra (chain complexes, homotopies, mapping cones, derived functors, Ext, Tor) and applications to various subjects (group theory: group extensions; commutative algebra: characterization of regular rings; geometry: sheaf cohomology and de Rham's theorem etc.). If there is enough time and interest, then I may discuss more advanced topics such as derived categories and spectral sequences. While I won't follow any book very closely, Weibel's book contains most of the topics that I'm likely to cover.

Book: C. Weibel, An introduction to homological algebra

Introduction in Algebraic Topology
Instructor: Professor Ralph Kaufmann
Course Number: MA 57200
Credits: Three
Time: 3:00–4:15 PM TTh

Description

The course is an introduction to algebraic topology. The focus will be on homology and cohomology theory which are a basic tool in many subjects. It is fundamental for topology, but also important for many other fields, such as differential, symplectic and algebraic geometry, number theory, mathematical physics, data science etc.

We will treat the classical simplicial and singular homology and cohomology, but we also plan to cover CW complexes and differential forms and more advanced topics as time permits.

The basic text for the course will be Elements of Algebraic Topology by James R. Munkres with additions from other sources and the lecture to update the material to a more modern presentation. These will be made available.

Arithmetic Harmonic Analysis: an introduction to the circle method
Instructor: Professor Trevor D. Wooley
Course Number: MA 59800AHA
Credits: Three
Time: 10:30–11:45 AM TTh

Description

This course serves as an introduction to the (Hardy-Littlewood) circle method, an important theme in analytic number theory, and complements the course proposed by Tess Anderson on discrete analysis – taking them in parallel, an option that is encouraged but not required, would enhance the level of understanding of each of them. Background results from number theory and harmonic analysis will be reviewed as needed.

The (Hardy-Littlewood) circle method applies Fourier analysis to count rational or integral solutions of an equation or inequality in a manner respecting the inherent arithmetic. Developments in recent years have broadened its impact into additive combinatorics and discrete harmonic analysis beyond its more traditional role in quantitative arithmetic geometry. A highlight from the past five years is the full resolution of the Main Conjecture in Vinogradov's mean value theorem.

We shall take as our central example Waring's problem – the problem of understanding the number of representations of an integer as the sum of a fixed number of k-th powers of positive integers.

Our aims are twofold: (i) to understand the scope and limitations of the circle method, and (ii) to gain some facility to apply the method, so from time to time there will be technical material that we'll just cite rather than prove in any detail. This course is intended to be accessible to those without any background in analytic number theory.

Assessment: Six (short) problem sets will be offered through the semester, and class participants can demonstrate engagement with the course by any written and/or in-class presentations featuring a reasonable subset of these problems – three levels of difficulty: short problems testing basic skill-sets, extended problems integrating the essential methods of the course, and more challenging problems for enthusiasts with detailed hints available on request.

Contents:

(i) Discussion of Weyl's inequality, Hua's Lemma, and the simplest treatment of Waring's problem. This provides an opportunity to discuss the key elements of the major arc analysis, that is, the singular integral and singular series, that together constitute the product of local densities. Density of integral zeros of diagonal equations.

(ii) Refinements to the major arc analysis, including use of Poisson summation. Sketch of Kloosterman method. Diminishing ranges. Diophantine equations arising as sums of binary forms.

(iii) Vinogradov's methods, especially Vinogradov's mean value theorem and ensuing analogue's of Weyl's inequality. Application to Waring's problem and diagonal equations. The Main Conjecture and its consequences.

iv) The Davenport-Heilbronn method (for establishing solubility of Diophantine inequalities). The Bentkus-Goetze-Freeman variant.

v) Unrepresentation theory – the theory of exceptional sets of integers that fail to be represented in a specified form.

The course will be based on the instructor's lecture notes. Good texts for background reading and support are:

R. C. Vaughan, The Hardy-Littlewood method, 2nd edn., Cambridge Tract No. 125, Cambridge University Press, 1997 [Condensed, but the best source in print; updated from the 1981 first edition.]

H. Davenport, Analytic methods for Diophantine equations and Diophantine inequalities, Ann Arbor Publishers, Ann Arbor, 1962 or the LaTeXed version published by Cambridge University Press in 2005 [Friendlier for the basics, with material on general homogeneous cubics, but misses modern developments.]

M. Nathanson, Additive number theory. The classical bases, GTM 164, Springer-Verlag, New York, 1996 [Pedestrian approach to the basics in which no corner is cut – good for getting started!]

Prerequisites: Elementary number theory and basic analysis.

The Discrete Jungle: Where Number Theory And Analysis Meet

Instructor: Professor Theresa Anderson
Course Number: MA 59800DJ
Credits: Three
Time: 9:00–10:15 AM TTh

Description

This course will introduce students to the fascinating interface of analysis and number theory through the world of discrete harmonic analysis. We will cover the basics, so only basic measure theory is assumed. Starting with some simple, yet powerful tools in analysis and elementary number theory results, such as those for exponential sums, we will both draw parallels between the fields as well as between the concurrent course proposed by Trevor Wooley in "Arithmetic Harmonic Analysis". With this dictionary in hand, we will begin by exploring fundamental operators from analysis in the continuous and discrete setting, and see how these regimes differ. Drastically different approaches are needed – we will focus on the discrete perspective and see how number theory comes into play as well as how information about discrete analogues have far reaching consequences.

An additional feature of this course is to develop presentation skills. The students will work either individually or in teams, depending on interest, to understand and present the main ideas from a recent research paper (or potentially another resource) in the area. Topics will be announced, and are flexible depending on interest. Additionally, I will be happy to consult with students about technical details.

We will rely on lecture notes and accessible versions of recent papers in the field, no text is needed. No number theory background is assumed. Ideal for graduate students interested in analysis, number theory, or related fields.

Prerequisites: Measure theory (basic, can be taken concurrently). Intended for graduate students in Mathematics. Recommended to take the complimentary course proposed by Trevor Wooley, but not required.

Higher Category Theory

Instructor: Professor Manuel Rivera
Course Number: MA 59800IC
Credits: Three
Time: 10:30–11:45 AM TTh

Description

Often in mathematics it is useful to replace "equality" or "isomorphism" with weaker notions of equivalence. For example, in topology we have the notions of homotopy between continuous maps and homotopy equivalence (or even weak homotopy equivalence) between spaces. In homological algebra we also encounter notions that behave similarly, like chain homotopy between chain maps, quasi-isomorphisms or chain equivalences between chain complexes.

Higher category theory is an abstract framework for describing constructions, usually inspired by homotopy theory, which are invariant under some notion of weak equivalence in a coherent sense. While an ordinary category has objects and morphisms a higher category has objects, 1-morphisms, 2-morphisms (morphisms between morphisms), etc... Another perspective is that a higher category theory encodes some "abstract homotopy theory" (a theory with constructions and results similar to those we encounter in the homotopy theory of spaces). There are different models to make sense of higher categories. Higher category theory has become increasingly important in different fields ranging from algebraic topology and geometry to theoretical physics and computer science.

The goal of this course is to develop the theory of higher categories based on quasi-categories, as studied in detail by Lurie and Joyal, with minimal prerequisites. We will also spend time developing the theory of model categories with the end goal of comparing quasi-categories to simplicial categories (categories enriched over simplicial sets), another model for higher category theory. We will assume basic knowledge of ordinary category theory and familiarity with the basic constructions and results of algebraic topology including the fundamental group and homology groups.

Large Deviations

Instructor: Professor Jonathon Peterson

Course Number: MA 59800LD

Credits: Three

Time: 9:30–10:20 AM MWF

Description

Large deviation theory is, informally, the study of the asymptotics of probabilities of unlikely events. As an example, while the law of large numbers says that the empirical mean of the sum of a large number of i.i.d. random variables is typically close to the expected value of the random variables, large deviation theory gives a framework for evaluating the asymptotics of the probability the empirical mean is close to a number different than the expected value. Large deviation theory not only gives understanding of how small probabilities of rare events are, but often also gives an insight into exactly how rare events arise (that is, given that a rare event occurred, what is the most likely way that this rare event happened). Large deviation theory is helpful for anyone working in probability theory but also has applications in statistics and other applied areas where one needs some understanding or control of rare events.

Prerequisites: 538 (532 and 539 will also be helpful but are not required).

Numerical Methods For PDEs I
Instructor: Professor Xiangxiong Zhang
Course Number: MA 61500
Credits: Three
Time: 10:30–11:20 AM MWF

Description

This is an introductory course of numerical solutions to partial differential equations for any graduate students interested in computational mathematics. The course will cover key concepts including accuracy, stability and convergence of finite difference methods for time dependent problems such as wave equations and parabolic equations. Linear system solvers such as the conjugate gradient method and the multigrid method, and ODE solvers such as Runge-Kutta method will also be discussed. As a special finite difference method, the finite difference implementation of finite element method on a rectangular grid will be introduced for solving the Poisson equation. If time permits, selected ad hoc topics including connections to optimization algorithms and applications to image processing will be briefly discussed. Homework and the final exam will consist of both analysis and coding by Matlab. Sample Matlab codes will be provided thus prior knowledge on coding is not required. Recommended prerequisites include linear partial differential equations, linear algebra and Fourier analysis, which will be reviewed during the lectures. Feel free to send an email to zhan1966@purdue.edu for questions. Last year's lecture notes are available at http://www.math.purdue.edu/∼zhan1966/teaching/615/MA615_notes.pdf

Class Field Theory
Instructor: Professor Freydoon Shahidi
Course Number: MA 68400
Credits: Three
Time: 9:30–10:20 AM MWF

Description

Class field theory is that of understanding abelian extensions of local and global fields. It is a crowning achievement of number theory in the 20th century and the main motivating object for the Langlands program. We will treat the subject by mainly concerntrating on number fields.

Syllabus: Ideles, adeles, L–functions, first and second inequalities, Artin symbol, reciprocity, local and global class fields, Kronecker–Weber theorem.

I will generally follow my notes which is now posted on my bio page. Other sources are:

1. S. Lang, "Algebraic Number Theory", Addison Wesley, 1970.
2. J. Cassels and A. Frolich, "Algebraic Number Theory", Thompson book company, 1967.

Prerequisite: MA 584 or the instructor's approval.

Introduction To p-adic Hodge Theory
Instructor: Professor Tong Liu
Course Number: MA 69000PHT
Credits: Three
Time: 1:30–2:20 PM MWF

Description

In this course, we will discuss the rudiments for p-adic Galois representations and p-adic Hodge theory, which is vital for the research of Langlands program from Galois and geometric side. I will discuss p-adic Galois representation of global Galois group, l-adic Galois representations, Weil-Deligne representation, construction of p-adic period rings, various p-adic representations (crystalline, semi-stable, de Rham, Hodge-Tate), and how these relate to various cohomology of algebraic variety.

Topics In Commutative Algebra: Homological Conjectures/Theorems And Perfectoid Spaces
Instructor: Professor Linquan Ma
Course Number: MA 69000PSHC
Credits: Three
Time: 1:30–2:45 PM TTh

Description

The goal of this course is to explain Hochster's direct summand conjecture/theorem and some of its consequences. This was proved by Yves Andre in 2016 using perfectoid algebras and spaces. To achieve this we will set up some foundations on almost mathematics and perfectoid space theory.

References:

1. Andre's papers: arXiv:1609.00320, arXiv:1609.00345
2. Bhatt and Heitmann–Ma's shorter proofs: arXiv:1608.08882, arXiv:1703.08281
3. Bhatt's lecture notes: http://www-personal.umich.edu/ bhattb/teaching/mat679w17/lectures.pdf
4. Scholze's thesis: https://www.math.uni-bonn.de/people/scholze/PerfectoidSpaces.pdf
5. Gabber–Ramero's book: arXiv:math/0201175

Analysis Of Hydrodynamic Equations In Complex Fluids
Instructor: Professor Changyou Wang
Course Number: MA 69300CW
Credits: Three
Time: 1:30–2:45 PM TTh

Description

This course will roughly cover three parts:

(1) Analysis of variational problems modeled on harmonic maps between Riemannian manifolds and their heat flows. Harmonic maps are critical points of Dirichlet energy functional between two manifolds, which are nonlinear extensions of harmonic functions and result in a quasilinear elliptic system with supercritical nonlinearity. I will mainly focus on the regularity issue of harmonic maps, along with some discussion on the structure of its singular sets. I will talk about harmonic map heat flows, which are negative gradient flow of the Dirichlet energy functional. I plan to discuss the work by Schoen-Uhlenbeck, Helein-Bethuel-Riviere, Eells-Sampson, and Chen-Struwe's.

(2) Basic theory on the incompressible Navier-Stokes equation, which is the governing equation for Newtonian fluids and constitutes the underlying equation for many complex fluids. Here I plan to discuss the existing regularity theory by Leray and Caffarelli-Kohn-Nirenberg.

(3) The static Oeen-Frank theory and the hydrodynamic Ericksen-Leslie theory on nematic liquid crystals. These equations model either minimal configurations or evolutions of the average orientation fields of nematic liquid crystal materials. Harmonic maps can be viewed as an approximation of critical points in the Oseen-Frank theory, while Ericksen-Leslie system can be viewed as a nonlinear coupling Between forced Navier-Stokes equations and transported harmonic map heat flows. I plan to discuss the work by Hardt-Lin-Kinderelehrer, and Lin-Wang.

Some familiarity with linear elliptic and parabolic PDEs equivalent to MATH 523 are preferable.

Topics In Complex Geometry
Instructor: Professor Sai Kee Yeung
Course Number: MA 69600A
Credits: Three
Time: 12:30–1:20 PM MWF

Description

Here are some tentative topics to be discussed, which may be adjusted as the class proceeds.

(a). Introduction to analytic aspects of zeta functions.

(b). Introduction to L^2 techniques in complex geometry.

(c). Introduction to Andre-Oort conjecture and related results.

(d). Introduction to some problems related to Abelian varieties.

Prerequisite: 562, 525

References: I would provide reference as the class proceeds.