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If $f(x) = \tan^{-1}\left(\frac{2}{x^2}\right)$, $f'(-1) =$

A. 1

B. $\frac{4}{5}$

C. $\frac{-2}{5}$

D. $\frac{2}{5}$

E. $\frac{-4}{5}$

$$\boxed{\frac{d}{dx}\tan^{-1}u = \frac{1}{1+u^2}\frac{du}{dx}}$$

$$\frac{d}{dx}\tan^{-1}\left(\frac{2}{x^2}\right) = \frac{1}{1+\left(\frac{2}{x^2}\right)^2} \cdot -\frac{4}{x^3}$$

$2x^{-2}$

at $x = -1$

$$= \frac{1}{1+\left(\frac{2}{1}\right)^2} \cdot -\frac{4}{(-1)^3} = \frac{1}{5} \cdot 4 = \frac{4}{5}$$

$$\boxed{\frac{d}{dx}\sin^{-1}u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}},$$

$$\boxed{\frac{d}{dx}\cos^{-1}u = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}}$$

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Suppose $f(x) = x^3 + x$. Find the slope of the tangent line to the graph of $y = f^{-1}(x)$ at the point $(2, 1)$.

point on $f^{-1}(x)$

A. 1/12

B. 1/10

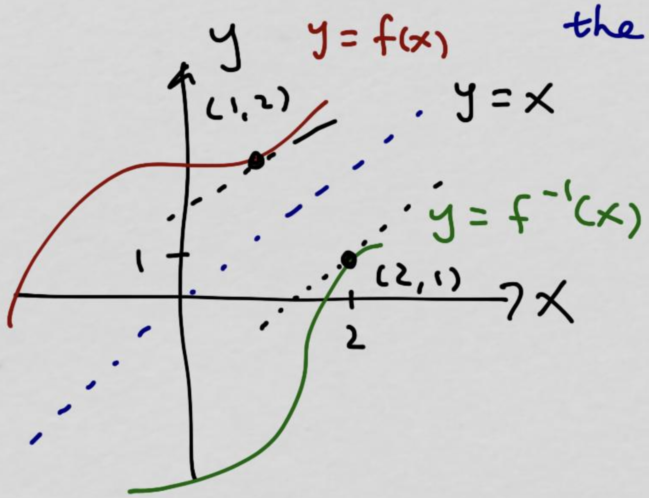
C. 1/2

D. 1/13

E. 1/4

$$\frac{d}{dx}\left[f^{-1}(x)\right] = \frac{1}{f'(y)}$$

note one uses x
the other uses y



$$\frac{d}{dx}\left[f^{-1}(2)\right] = \frac{1}{f'(1)}$$

corresponding y-value to $x = 2$

$f(x) = x^3 + x$

$f'(x) = 3x^2 + 1$

$f'(1) = 4$

so, $\frac{d}{dx}\left[f^{-1}(2)\right] = \frac{1}{f'(1)} = \boxed{\frac{1}{4}}$

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If R the total resistance across a circuit is given by $1/R = 1/R_1 + 1/R_2$ for two resistors with resistances R_1 and R_2, how fast is the total resistance changing when $R_1 = 5, R_2 = 10$ and R_1 is changing at 1 ohm/s and R_2 is changing at 2 ohm/s?

A. 2/3 ohm/s

B. 5/7 ohm/s

C. 9/2 ohm/s

D. 4/5 ohm/s

E. 4/3 ohm/s

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$ find $\frac{dR}{dt}$ when $R_1 = 5$, $R_2 = 10$, $\frac{dR_1}{dt} = 1$, $\frac{dR_2}{dt} = 2$

differentiate implicitly with respect to t

$$-\frac{1}{R^2}\frac{dR}{dt} = -\frac{1}{R_1^2}\frac{dR_1}{dt} - \frac{1}{R_2^2}\frac{dR_2}{dt}$$

now we can plug in numbers

$$-\frac{1}{R^2}\frac{dR}{dt} = -\frac{1}{25}(1) - \frac{1}{100}(2)$$

find R from

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{5} + \frac{1}{10} = \frac{3}{10}$$

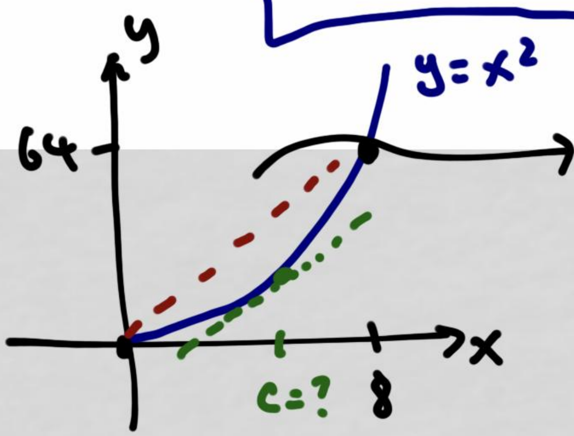
$$R = 10/3$$

$$\frac{dR}{dt} = \frac{100}{9}\left(\frac{3}{50}\right) = \boxed{\frac{2}{3}}$$

Find the number c that satisfies the conclusion of the Mean Value Theorem for the function $f(x) = x^2$ on the interval $[0, 8]$ (that is, $a = 0$ and $b = 8$).

A. 2
B. 3
C. 4
D. 5
E. 6

MVT : if $f(x)$ is continuous on $[a,b]$
and $f(x)$ is differentiable on (a,b)
then there is at least c, $a < c < b$
where $f'(c) = \dfrac{f(b) - f(a)}{b - a}$



slope : $\dfrac{f(8) - f(0)}{8 - 0} = \dfrac{64}{8} = 8$

$f'(x) = 2x = 8$

$x = 4$

where on $f(x)$ is tangent line slope $= 8$?

Rolle's Theorem

$f(x)$ is continuous on $[a,b]$
$f(x)$ is differentiable on (a,b)
$f(b) = f(a)$

if true, then there is at least one c, $a < c < b$ where $f'(c) = 0$

The derivative of a function g is $g'(x) = \sin x - \sin 2x$, so that $x = 0$ and $x = \pi/3$ are critical numbers of g. Then, g has

A. a local minimum at 0 and a local maximum at $\pi/3$

B. a local maximum at 0 and a local minimum at $\pi/3$

C. a local maximum at 0 and an inflection point at $\pi/3$

D. a local maximum at $\pi/3$

E. inflection points at $0, \pi/3$

$g' = 0 \rightarrow x = 0, \ x = \pi/3$

g': $+ +$ (↗) $\quad 0 \quad$ $- -$ (↘) $\quad 0 \quad$ $+ +$ (↗)

$x = 0 \qquad x = \pi/3$

relative max at $x = 0$ $(+ \rightarrow -)$

relative min at $x = \pi/3$ $(- \rightarrow +)$

or, use the Second Deriv. Test

$g'' = \cos x - 2\cos 2x$

$g''(0) = 1 - 2 < 0$ max

$g''\left(\frac{\pi}{3}\right) = \frac{1}{2} - 2\left(-\frac{1}{2}\right) > 0$ min

How many inflection points does the graph of $y = \frac{1}{12}x^4 - \frac{1}{3}x^3 + \frac{1}{2}x^2$ have?

↳ where f'' changes sign

$y' = \frac{1}{3}x^3 - x^2 + x$

$y'' = x^2 - 2x + 1$

solve $y'' = 0 \rightarrow (x-1)(x-1) = 0 \rightarrow x = 1$

NOT necessarily where the inflection pt is

A. 0
B. 1
C. 2
D. 3
E. 4

y'' + + | 0 | + +

$x = 1$

$y''(0) = 1$

$y''(2) = 4 - 4 + 1 = 1$

no sign change, so no inflections

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Let $f(x) = x^4 - 8x^2$ for $-1 \le x \le 3$. If m_1 is the absolute maximum of f and m_2 is the absolute minimum find $m_1 + m_2$.

A. 2
B. 9
C. -7
D. -16
E. 0

find critical numbers ($f'=0$ and f' DNE)

compare $f(x)$ at the critical numbers and end points

$f' = 4x^3 - 16x = 0$

outside $-1 \le x \le 3$

$4x(x^2-4) = 0 \rightarrow x=0,\ \cancel{x=-2},\ x=2$

original function

$f(-1) = 1-8 = -7$

$f(0) = 0$

$f(2) = 16-32 = -16 \rightarrow$ abs. min $= m_2$

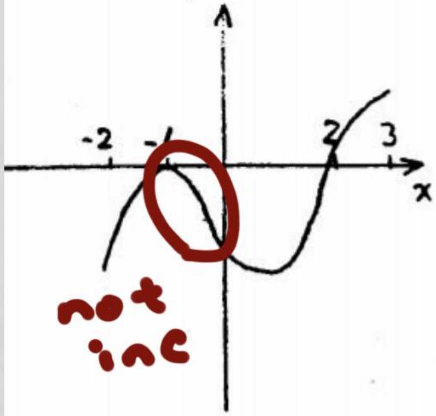
$f(3) = 81-72 = 9 \rightarrow$ abs. max $= m_1$

$m_1 + m_2 = 9 - 16 = -7$

Given that $f'(x) > 0$ when $-1 < x < 0$ and $2 < x < 3$, and $f'(x) < 0$ when $-2 < x < -1$ and $0 < x < 2$, which could be the graph of f?

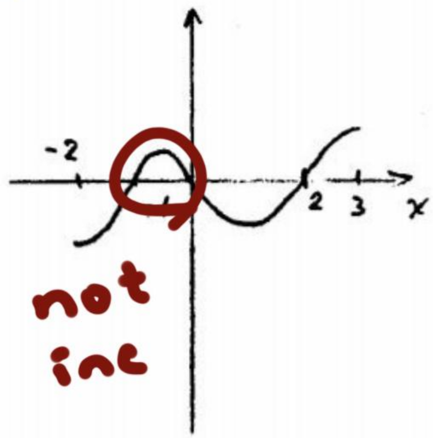
inc: $(-1, 0)$, $(2, 3)$

dec: $(-2, -1)$, $(0, 2)$



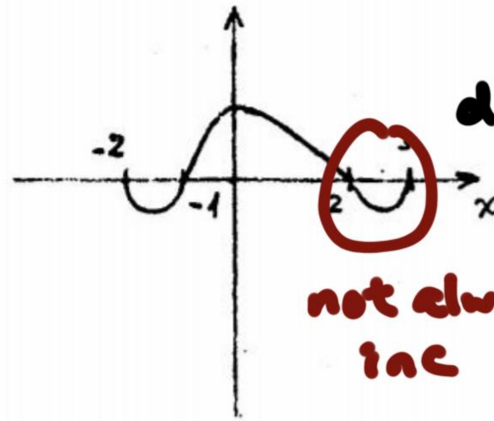
not inc

X



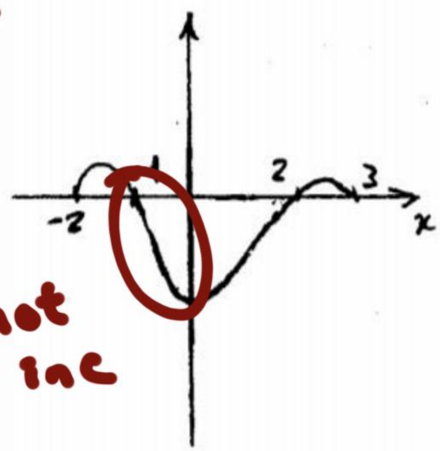
not inc

X



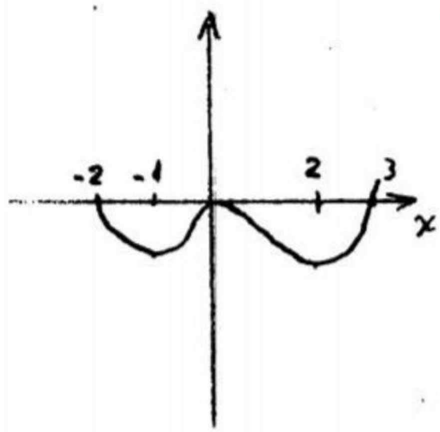
not always inc

X



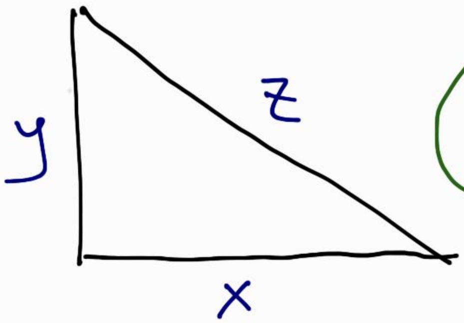
not inc

X



E.

If the sum of the length of the legs of a right triangle is 7, what is the minimum length of its hypotenuse?



$x + y = 7$ constraint

minimize z objective

$y = 7 - x$

A. 8

B. 7

C. 5

D. $5\sqrt{2}$

E. $\dfrac{7\sqrt{2}}{2}$

$$z^2 = x^2 + y^2$$

$$z = \sqrt{x^2 + y^2}$$

eliminate y

$$z = \sqrt{x^2 + (7-x)^2}$$

$$z = (2x^2 - 14x + 49)^{1/2}$$

$$0 \le x \le 7$$

$$z' = \frac{1}{2}(2x^2 - 14x + 49)^{-1/2}(4x - 14)$$

$$= \frac{2x - 7}{\sqrt{2x^2 - 14x + 49}}$$

$z' = 0 \rightarrow 2x - 7 = 0 \rightarrow x = 7/2$

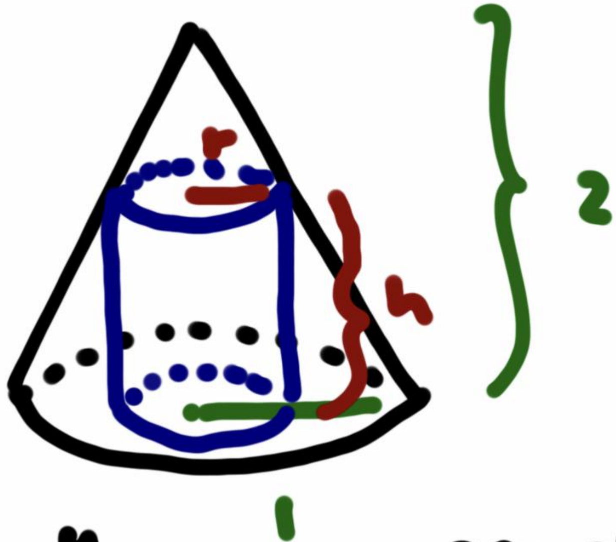
$\frac{1}{\sqrt{2}} = \frac{1}{1.4}$

$z(0) = 7$

$z(7/2) = \sqrt{\left(\frac{7}{2}\right)^2 + \left(\frac{7}{2}\right)^2} = \sqrt{\frac{98}{4}} = \frac{7\sqrt{2}}{2}$ min

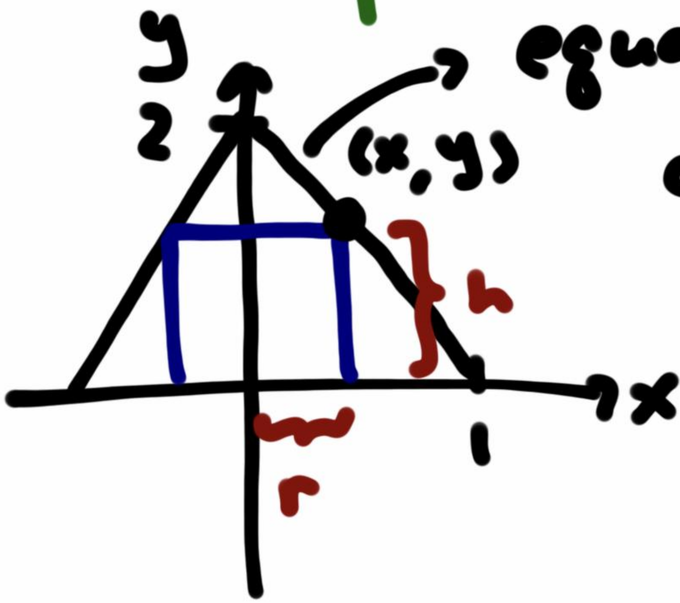
$z(7) = 7$

Find the maximum possible volume of a cylinder that is inscribed in a cone of diameter 2 and height 2.



cylinder volume

$= \pi r^2 h$



equation: $y = 2 - 2x$

corner on the line

$(x, y) \to (x, 2-2x)$

$r = x$

$h = y$

A. $\frac{8\pi}{9}$

B. $\frac{4\pi}{9}$

C. $\frac{8\pi}{27}$

D. $\frac{4\pi}{27}$

E. $\frac{4\pi}{3}$

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Volume of cylinder: $V = \pi r^2 h = \pi x^2 y$

eliminate y: corner is $(x, y) = (x, 2-2x)$

$$V = \pi x^2 (2-2x) \qquad 0 \le x \le 1$$

$$V = 2\pi x^2 - 2\pi x^3$$

$$V' = 4\pi x - 6\pi x^2 = 0$$

$$2\pi x (2-3x) = 0$$

$$x = 0, \quad x = 2/3$$

$$V(0) = 0$$

$$V(2/3) = \pi \left(\frac{2}{3}\right)^2 \left(2 - \frac{4}{3}\right) = \pi \left(\frac{4}{9}\right)\left(\frac{2}{3}\right) = \boxed{\frac{8\pi}{27}} \quad \text{max}$$

$$V(1) = 0$$

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