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Spring 2019

2. Which of the following has a removable discontinuity at $x = -3$?

↳ at x where BOTH the denominator and numerator are zero

A. $f(x) = \dfrac{x^2-9}{x-3}$ $\rightarrow$ $f(x) = \dfrac{(x+3)(x-3)}{x-3}$ not $\frac{0}{0}$ at $x=-3$

B. ~~$f(x) = \dfrac{1}{\sqrt{x+3}}$~~

C. $f(x) = \dfrac{x^2-9}{x+3}$ $\rightarrow$ $f(x) = \dfrac{(\cancel{x+3})(x-3)}{\cancel{x+3}}$ is $\frac{0}{0}$ at $x=-3$

$= x-3$ when $x \neq -3$

removable discontinuity is NOT an asymptote

asymptote → numerator $\neq 0$ while the denominator is

D. ~~$\ln(x+3)$~~

E. ~~$\sqrt[3]{x+3}$~~

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5. Which of the following is TRUE?

I. $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) = \frac{\pi}{3}$ ✓

II. $\cos^{-1}\left(\cos\left(\frac{5\pi}{4}\right)\right) = -\frac{\pi}{4}$ X

III. $\csc\left(\tan^{-1}\left(\frac{1}{x}\right)\right) = \sqrt{x^2+1}$ ✓

A. I only

B. II only

C. III only

D. I and II only

E. I and III only

I. $\sin^{-1}\left(\underbrace{\sin\left(\frac{2\pi}{3}\right)}_{x}\right) = \frac{\pi}{3}$

$x = \sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$\underbrace{\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)}_{\text{find angle in } [-\frac{\pi}{2}, \frac{\pi}{2}] \text{ where } \sin(\theta) = \frac{\sqrt{3}}{2}}$

$\sin^{-1}(x)$ has range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

find angle in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ where $\sin(\theta) = \frac{\sqrt{3}}{2}$

$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

↳ is in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

so, $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) = \frac{\pi}{3}$ is true

II. $\cos^{-1}\left(\underbrace{\cos\left(\frac{5\pi}{4}\right)}_{x = \cos\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}}\right) = -\frac{\pi}{4}$

$\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ range of $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ is $[0, \pi]$

find an angle θ in $[0, \pi]$ where $\cos(\theta) = -\frac{\sqrt{2}}{2}$

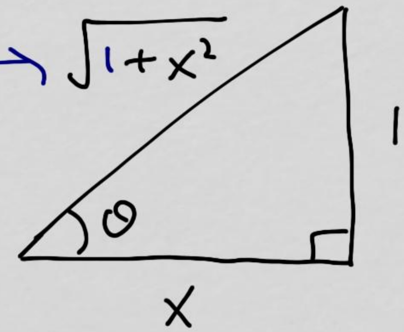
$\theta = \frac{3\pi}{4}$

so, $\cos^{-1}\left(\cos\left(\frac{5\pi}{4}\right)\right) = \frac{3\pi}{4}$ and not $-\frac{\pi}{4}$

II is false

III. $\csc\left(\underbrace{\tan^{-1}\left(\frac{1}{x}\right)}_{\theta = \tan^{-1}\left(\frac{1}{x}\right)}\right) = \sqrt{x^2+1}$

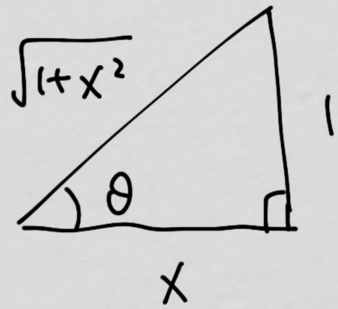
$\tan(\theta) = \frac{1}{x} = \frac{\text{opp}}{\text{adj}}$



$$\csc\left(\underbrace{\tan^{-1}\left(\tfrac{1}{x}\right)}\right)$$

$$\theta = \tan^{-1}\left(\tfrac{1}{x}\right)$$

$$\tan(\theta) = \frac{1}{x}$$



$$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{1}{\frac{opp}{hyp}} = \frac{hyp}{opp} = \frac{\sqrt{1+x^2}}{1} = \sqrt{1+x^2}$$

so, $\csc\left(\tan^{-1}\left(\frac{1}{x}\right)\right) = \sqrt{1+x^2}$ <u>is</u> true

7. If

$$\tan y = \frac{2}{x^2} \rightarrow 2x^{-2}$$

find y' when $x = -1$

A. 1

B. $\frac{4}{5}$

C. $\frac{-2}{5}$

D. $\frac{2}{5}$

E. $\frac{-4}{5}$

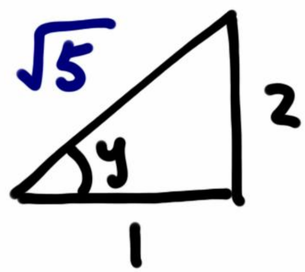
differentiate implicitly

$$\sec^2 y \cdot y' = -4x^{-3}$$

$$y' = -\frac{4}{x^3} \cdot \frac{1}{\sec^2 y} = -\frac{4}{x^3} \cdot \cos^2 y$$

find y when $x = -1$

$$\tan y = \frac{2}{x^2} = \frac{2}{1} \rightarrow$$



$$\cos y = \frac{\text{adj}}{\text{hyp}} = \frac{1}{\sqrt{5}}$$

$$y' = -\frac{4}{(-1)^3} \cdot \left(\frac{1}{\sqrt{5}}\right)^2 = 4 \cdot \frac{1}{5} = \frac{4}{5}$$

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11. If a certain radioactive substance has a half-life of 7 days, how long, in days, does it take for the sample to decay to $\frac{1}{3}$ of its original amount?

exponential growth / decay eq : $\boxed{y(t) = y_0 e^{kt}}$

half-life : t when $y(t) = \frac{1}{2} y_0$ sub into

$\frac{1}{2} \cancel{y_0} = \cancel{y_0} e^{k \cdot 7}$

$\frac{1}{2} = e^{7k}$

$\ln \frac{1}{2} = 7k$

$\boxed{k = \frac{1}{7} \ln \frac{1}{2}}$

A. $\dfrac{\ln 14}{\ln 2}$

B. $\dfrac{3 \ln 7}{\ln 2}$

C. $\dfrac{7 \ln 3}{\ln 2}$

D. $\dfrac{3 \ln 2}{\ln 7}$

E. $\dfrac{7 \ln 2}{\ln 3}$

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time to decay to $\frac{1}{3}y_0$: $y(t) = y_0 e^{kt}$ $k = \frac{1}{7}\ln\frac{1}{2}$

$$\frac{1}{3}\cancel{y_0} = \cancel{y_0}\, e^{\left(\frac{1}{7}\ln\frac{1}{2}\right)t}$$

$$\frac{1}{3} = e^{\left(\frac{1}{7}\ln\frac{1}{2}\right)t}$$

$$\ln\frac{1}{3} = \left(\frac{1}{7}\ln\frac{1}{2}\right)t$$

$$t = \frac{7\cdot\ln\frac{1}{3}}{\ln\frac{1}{2}}$$

to match up with the choices, use the fact $\ln\frac{1}{3} = \ln 3^{-1} = -\ln 3$

$\ln\frac{1}{2} = \ln 2^{-1} = -\ln 2$

$$t = \frac{7\cdot -\ln 3}{-\ln 2} = \boxed{\frac{7\ln 3}{\ln 2}}$$

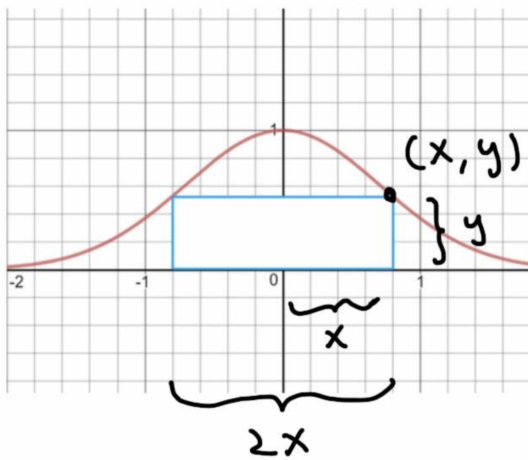
18. Find the area of the largest rectangle that can be inscribed under the curve $y = e^{-x^2}$ and above the x-axis.

area of rectangle: $A = 2xy$

$A = 2xe^{-x^2}$

find x that maximizes A

(then find y and A)



product rule

$$A' = 2(x)(e^{-x^2} \cdot -2x) + 2(e^{-x^2})(1)$$

$$= \underbrace{(2e^{-x^2})}_{\text{exponential is never zero}} \underbrace{(-2x^2+1) = 0}_{\text{this is the only place to find critical numbers}}$$

$$-2x^2 + 1 = 0$$

A. $\sqrt{\frac{2}{e}}$

B. $\sqrt{2e}$

C. $\frac{2}{e}$

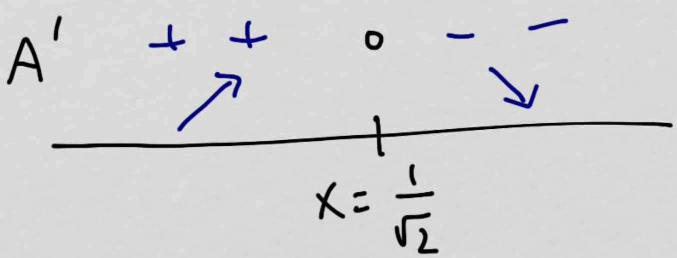
D. $\frac{1}{\sqrt{2e}}$

E. $\frac{2}{e^2}$

$$x^2 = \frac{1}{2}$$

$$x = \frac{1}{\sqrt{2}},\ \cancel{-\frac{1}{\sqrt{2}}}$$

x is half width, which cannot be negative



$$A' = \underbrace{2e^{-x^2}}_{\text{always positive}} \underbrace{(-2x^2+1)}_{\text{responsible for sign change}}$$

so A is a maximum when $x = \frac{1}{\sqrt{2}}$

problem wants the maximum area : $A = 2xy \qquad y = e^{-x^2}$

$$A = 2\left(\frac{1}{\sqrt{2}}\right)e^{-\left(\frac{1}{\sqrt{2}}\right)^2}$$

$$= \frac{2}{\sqrt{2}}e^{-1/2} = \sqrt{2}\,e^{-1/2} = \frac{\sqrt{2}}{\sqrt{e}} = \boxed{\sqrt{\frac{2}{e}}}$$

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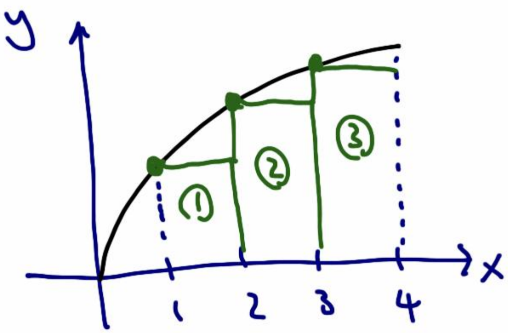
19. Use a Riemann Sum to **estimate** the area under the graph of $f(x) = \sqrt{x}$ from $x = 1$ to $x = 4$ using **three** approximating rectangles and **left** endpoints.



A. $\sqrt{1} + \sqrt{2} + \sqrt{3}$

B. $\sqrt{2} + \sqrt{3} + \sqrt{4}$

C. $\sqrt{1} + \sqrt{2} + \sqrt{3} + \sqrt{4}$

D. $\frac{1}{3}(\sqrt{1} + \sqrt{2} + \sqrt{3})$

E. $\frac{1}{3}(\sqrt{2} + \sqrt{3} + \sqrt{4})$

each has width of 1

height of 1 : $f(1) = \sqrt{1} = 1$

" " 2 : $f(2) = \sqrt{2}$

" " 3 : $f(3) = \sqrt{3}$

estimate of area : $\underbrace{(1)(1)}_{1} + \underbrace{(1)(\sqrt{2})}_{2} + \underbrace{(1)(\sqrt{3})}_{3}$

$= 1 + \sqrt{2} + \sqrt{3} = \boxed{\sqrt{1} + \sqrt{2} + \sqrt{3}}$

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20. If $y(x) = \int_3^{\tan(x)} \sqrt{\sqrt{t} + 6t}\, dt$, use Part 1 of the Fundamental Theorem of Calculus to find $y'(\frac{\pi}{4})$.

$$\boxed{\text{FTC 1}: \ \frac{d}{dx}\int_a^{x} f(t)\,dt = f(x)}$$ must be just x

need Chain Rule here

let $u = \tan(x)$

then $y(u) = \int_3^u \sqrt{\sqrt{t}+6t}\,dt$

and by Chain Rule, $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

A. $\sqrt{7}$

B. $2\sqrt{7}$

C. $\sqrt{\frac{3\pi}{2} + \sqrt{\frac{\pi}{4}}}$

D. $2\sqrt{\frac{3\pi}{2} + \cancel{\sqrt{\frac{\pi}{4}}}}$?

E. 2

$$y' = \underbrace{\left(\frac{d}{du}\int_3^u \sqrt{\sqrt{t}+6t}\right)}_{\text{FTC 1}} \cdot \left(\frac{d}{dx}\cdot \tan x\right)$$

$$= \sqrt{\sqrt{u}+6u} \cdot \sec^2(x)$$

$$y' = \sqrt{\sqrt{u} + 6u} \cdot \sec^2(x) \qquad u = \tan(x)$$

$$y'\left(\frac{\pi}{4}\right) = \sqrt{\sqrt{\tan\frac{\pi}{4}} + 6\cdot\frac{\pi}{4}} \cdot \sec^2\left(\frac{\pi}{4}\right)$$

should be $\tan\frac{\pi}{4} = 1$

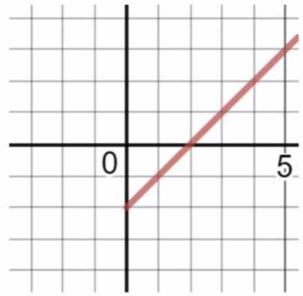
$$= \sqrt{1 + \cancel{\frac{6\pi}{4}}\,6} \cdot \frac{1}{\left(\cos\left(\frac{\pi}{4}\right)\right)^2} = \sqrt{1 + \cancel{\frac{6\pi}{4}}\,6} \cdot \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2}$$

$$= \sqrt{1 + \cancel{\frac{6\pi}{4}}\,6} \cdot \frac{4}{2} = 2\sqrt{1 + \cancel{\frac{6\pi}{4}}\,6} = 2\sqrt{7}$$

~~closest answer is~~ D

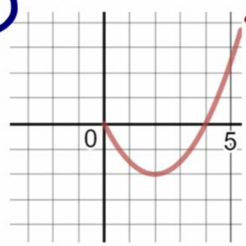
~~(typo on exam?)~~

25.

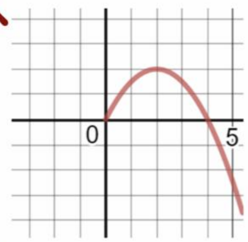


The figure above shows the graph of $f(x)$. Which of the graphs below is $\int_0^x f(t)\,dt$?

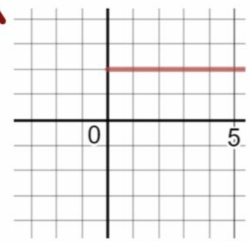
A



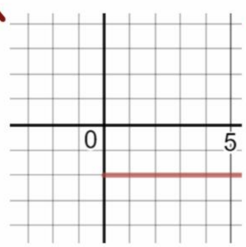
B



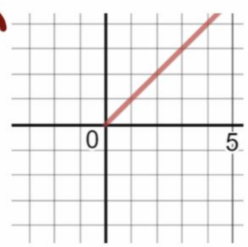
C



D



E



one way: use FTC 1

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

so, $$\underbrace{\frac{d}{dx}\underbrace{\int_0^x f(t)\,dt}_{\text{graph we want find}}}_{\text{tangent line of graph we want}} = \underbrace{f(x)}_{\text{given graph}}$$

so, we want the tangent slope at x on the graph we want to be equal to $f(x)$ value on the given graph

at $x=2$, $f(2)=0$ (given graph)

this eliminates E.

at $x=0$, $f(0)=-2$ (given graph)

we want the graph we want to have tangent slope $=-2$ at $x=0$

B is eliminated because slope at 0 is positive, C and D are eliminated because their slopes at 0 at $x=0$

The second way to do it is to interpret $\int_0^x f(t)\,dt$ as the net area between the x-axis and the given $f(x)$

we can see from the top graph that $\int_0^2 f(t)\,dt = -2$ (triangle below x-axis totaling in area of 2 squares)

so, since $\int_0^x f(t)\,dt$ is the function whose graph we want, we want to pick the one where $y=-2$ at $x=2$ (since $\int_0^{-2} f(t)\,dt = 2$)

this rules out B, C, E $\to$ all are positive at $x=2$

check one more x: $\int_0^4 f(t)\,dt = 0$ since it has the same triangle above and below x-axis, so net area is 0

therefore, we want to pick the graph where $y = 0$ at $x = 4$

that leaves only A.