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8. Find the limit.

F2018

$$\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{4x} \to 1^{\infty} \quad \text{indeterminate form}$$

transform into $\frac{0}{0}$ or $\frac{\infty}{\infty}$ then use L'Hospital's Rule

A. e^4

B. 4

C. e^{12}

D. 12

E. $e^{3/4}$

let $y = \left(1+\frac{3}{x}\right)^{4x}$ we want $\lim_{x\to\infty} y$

$$\ln y = \ln\left(1+\frac{3}{x}\right)^{4x}$$

$$= (4x)\ln\left(1+\frac{3}{x}\right) = \frac{\ln\left(1+\frac{3}{x}\right)}{\frac{1}{4x}}$$

multiply by $4x$ is same as divide by $\frac{1}{4x}$

$$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\ln\left(1+\frac{3}{x}\right)}{\frac{1}{4x}} \to \frac{0}{0}$$

L'Hospital's Rule ok

$$\overset{L}{=} \lim_{x\to\infty} \frac{\frac{1}{1+\frac{3}{x}} \cdot \frac{d}{dx}\left(1+\frac{3}{x}\right)}{\frac{1}{4}\left(-\frac{1}{x^2}\right)}$$

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$$= \lim_{x\to\infty} \frac{\frac{1}{1+\frac{3}{x}}\left(-\frac{3}{x^2}\right)}{-\frac{1}{4x^2}} \cdot \frac{-4x^2}{-4x^2}$$

$$= \lim_{x\to\infty} \frac{\frac{12}{1+3/x}}{1} = \frac{12}{1} = 12 = \lim_{x\to\infty} \ln y$$

we want $\lim_{x\to\infty} y$

$$\lim_{x\to\infty} \ln y = 12 \longrightarrow \ln y = 12 \longleftrightarrow y = e^{12}$$

$$\lim_{x\to\infty} y = \boxed{e^{12}}$$

15. Water is poured into a conical paper cup at the rate of 4 cubic centimeters per second. If the cup is 18 cm tall and the top has a radius of 6 cm, how fast is the water level rising when the water is 9 cm deep? (Volume of the cone: $V = \frac{1}{3}\pi r^2 h$).

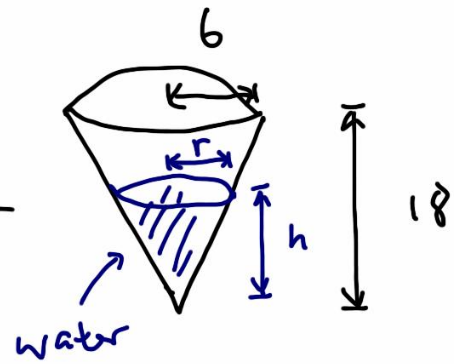
A. $\frac{4}{9\pi}$ cm/s

B. $\frac{\pi}{3}$ cm/s

C. $\frac{4\pi}{9}$ cm/s

D. $\frac{4}{81\pi}$ cm/s

E. $\frac{9\pi}{4}$ cm/s



$\frac{dV}{dt} = 4$

don't use until after derivative

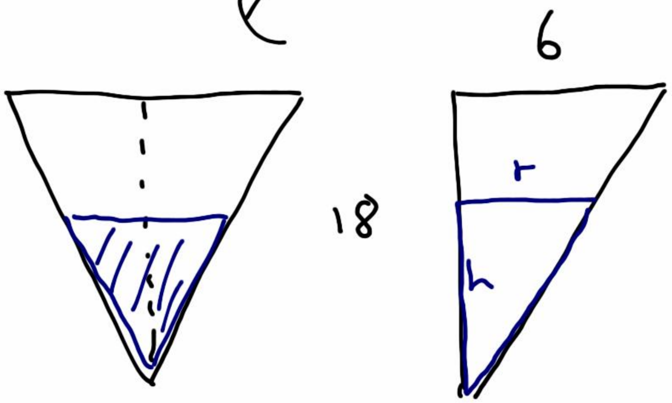
find $\frac{dh}{dt}$ when $h = 9$

$V = \frac{1}{3}\pi r^2 h$

has two variables : r, h

need to get rid of one of them

keep h because we want $\frac{dh}{dt}$



by similar triangles,

$\frac{r}{h} = \frac{6}{18} \rightarrow r = \frac{1}{3}h$ sub into V

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$V = \frac{1}{3}\pi r^2 h$ and $r = \frac{1}{3}h$

$= \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h = \frac{1}{3}\pi \cdot \frac{1}{9}h^3 = \frac{1}{27}\pi h^3$

differentiate with respect to t

$$\frac{dv}{dt} = \frac{1}{27}(3\pi h^2)\frac{dh}{dt}$$

$$= \frac{1}{9}\pi h^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{9 \cdot \frac{dv}{dt}}{\pi h^2}$$

now sub in $\frac{dv}{dt} = 4$, $h = 9$

$$= \frac{9 \cdot 4}{\pi \cdot 81} = \boxed{\frac{4}{9\pi}}$$

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16. By linearization (differentials), the approximate value of $\sqrt[4]{17}$ is

A. 2

B. $\frac{63}{32}$

C. $\frac{31}{16}$

D. $\frac{33}{16}$

E. $\frac{65}{32}$

$f(x) = \sqrt[4]{x} = x^{1/4}$

the nearest convenient number to 17 whose 4^{th} root is easy to find is 16 $\rightarrow a = 16$

linearization near $a = 16$: $\boxed{L(x) = f(a) + f'(a)(x-a)}$

eq. of tangent line at $x = a$

$f(a) = \sqrt[4]{16} = 2$

$f'(x) = \frac{1}{4} x^{-3/4} = \frac{1}{4x^{3/4}}$ $\quad f'(a) = f'(16) = \frac{1}{4 \cdot (16)^{3/4}}$

$= \frac{1}{4 \cdot (16^{1/4})^3} = \frac{1}{4 \cdot 2^3}$

$= \frac{1}{32}$

$$L(x) = f(a) + f'(a)(x-a)$$

$$L(x) = 2 + \frac{1}{32}(x-16) \approx \sqrt[4]{x}$$

$$\sqrt[4]{17} \approx 2 + \frac{1}{32}(17-16) \approx 2 + \frac{1}{32} = \boxed{\frac{65}{32}}$$

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20. Find an antiderivative of the function $f(x) = \dfrac{(x-1)^2}{x}$

A. $\dfrac{2(x-1)^3}{3x^2}$

B. $\dfrac{x^2-1}{x^2}$

C. $\dfrac{1}{2}x^2 - 2x + \ln|x|$

D. $\dfrac{x(x-1)^3}{3}$

E. $\dfrac{(x-1)^2(2x+1)}{6}$

try simple things first before substitution

$$f(x) = \frac{(x-1)^2}{x} = \frac{x^2-2x+1}{x} = x-2+\frac{1}{x}$$

$$\int \left(x-2+\frac{1}{x}\right)dx = \frac{x^2}{2} - 2x + \ln|x| + C$$

could be any constant including zero (that's why the choices don't have the C)

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25. $\int_0^\pi \sin t\,\sqrt{1+\cos t}\,dt =$

A. $-\dfrac{2\pi\sqrt{\pi}}{3}$

B. $\dfrac{4\sqrt{2}}{3}$

C. $\dfrac{4}{3}$

D. 0

E. $-\sqrt{\pi}$

$$\int_0^\pi (\sin t)\,(1+\cos t)^{1/2}\,dt$$

$\dfrac{d}{dt}(1+\cos t) = -\sin t$ which is a constant multiple of the other part

so, $u = 1+\cos t$

$\dfrac{du}{dt} = -\sin t \rightarrow du = -\sin t\,dt$

$\sin t\,dt = -du$

old upper limit: $t=\pi \rightarrow u = 1+\cos(\pi) = 0$

old lower limit: $t = 0 \rightarrow u = 1+\cos(0) = 2$

$$\int_2^0 -u^{1/2}\,du = -\int_2^0 u^{1/2}\,du = -\left.\frac{u^{3/2}}{3/2}\right|_2^0$$

Do NOT change back to x

$$= -\left.\frac{2}{3}u^{3/2}\right|_2^0 = -\frac{2}{3}(0)^{3/2} - -\frac{2}{3}(2)^{3/2} = \frac{2}{3}(2)^{3/2} = \frac{2}{3}\cdot 2\cdot\sqrt{2} = \boxed{\frac{4\sqrt{2}}{3}}$$

F2017

9. Suppose that $f(x)$ and $g(x)$ are functions with $f(1) = 2$, $f'(1) = 9$, $g(1) = 2$ and $g'(1) = 4$. Let $h(x) = \dfrac{f(e^{2x})}{g(e^{3x})}$. Find $h'(0)$.

A. 1

B. 2

C. 3

D. 4

E. 5

$$h(x) = \frac{f(e^{2x})}{g(e^{3x})}$$

take deriv, using quotient rule

remember $\frac{d}{dx} f(u) = f'(u)\frac{du}{dx}$

$$h'(x) = \frac{g(e^{3x}) \cdot \frac{d}{dx} f(e^{2x}) - f(e^{2x}) \cdot \frac{d}{dx} g(e^{3x})}{[g(e^{3x})]^2}$$

deriv. of e^{2x}

$$h'(x) = \frac{g(e^{3x}) \cdot f'(e^{2x}) \cdot e^{2x} \cdot 2 - f(e^{2x}) \cdot g'(e^{3x}) \cdot e^{3x} \cdot 3}{[g(e^{3x})]^2}$$

$$h'(0) = \frac{g(1) \cdot f'(1) \cdot 2 - f(1) \cdot g'(1) \cdot 3}{[g(1)]^2} = \frac{2 \cdot 9 \cdot 2 - 2 \cdot 4 \cdot 3}{2^2} = \boxed{3}$$

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14. The Mean Value Theorem guarantees that the derivative of $f(x) = \sqrt{1+x^3}$ at some point on the interval $(0, 2)$ is

MVT: if $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b), then at some point c, $a < c < b$, $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

here, $\dfrac{f(b) - f(a)}{b - a} = \dfrac{\sqrt{1+8} - \sqrt{1+0}}{2 - 0} = \dfrac{3-1}{2} = 1$

so, somewhere between 0 and 2, $f'(x) = 1$

A. 0

B. 1

C. 2

D. 3

E. 4

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18. If $f''(x) = 2x$ and $f(0) = 4$, $f'(0) = -3$, find $f(3)$

A. 0

B. 2

C. 4

D. 7

E. 9

undo differentiation twice to find $f(x)$ from $f''(x)$

$$f'(x) = \int f''(x)\,dx = \int 2x\,dx = x^2 + C$$

find C using $f'(0) = -3$

$$-3 = (0)^2 + C \rightarrow C = -3$$

so, $\boxed{f'(x) = x^2 - 3}$

again:

$$f(x) = \int f'(x)\,dx = \int (x^2 - 3)\,dx = \frac{x^3}{3} - 3x + D$$

find D using $f(0) = 4$

$$4 = \frac{0^3}{3} - 3(0) + D \rightarrow D = 4$$

so, $\boxed{f(x) = \frac{1}{3}x^3 - 3x + 4}$ $\quad f(3) = \frac{27}{3} - 9 + 4 = \boxed{4}$

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