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Practice Problems

16. Suppose that the mass of a radioactive substance decays from 18 gms to 2 gms in 2 days. How long will it take for 12 gms of this substance to decay to 4 gms? A. $\frac{\ln 3}{\ln 2}$ days B. 1 day C. $\frac{\ln 2}{\ln 3}$ days D. 2 days E. $(\ln 3)^2$ days

exponential decay $\rightarrow$ $\boxed{y(t) = y_0 e^{kt}}$

find k: $y_0 = 18$, $y(2) = 2$

2 days (t in days)

$$2 = 18 e^{k \cdot 2}$$

$$\frac{2}{18} = e^{2k} \rightarrow \frac{1}{9} = e^{2k} \rightarrow \ln \frac{1}{9} = 2k \rightarrow \boxed{k = \frac{1}{2} \ln \frac{1}{9}}$$

If started with 12, how long to decay to 4?

$$4 = 12 e^{\left(\frac{1}{2}\ln\frac{1}{9}\right)t}$$

$$\frac{1}{3} = e^{\left(\frac{1}{2}\ln\frac{1}{9}\right)t} \rightarrow \ln\frac{1}{3} = \left(\frac{1}{2}\ln\frac{1}{9}\right)t \rightarrow$$

$$t = \frac{2 \ln \frac{1}{3}}{\ln \frac{1}{9}}$$

$\rightarrow \ln 3^{-1} = -\ln 3$

$\rightarrow -\ln 9 = -\ln 3^2 = -2\ln 3$

$$t = \frac{2 \cdot -\ln 3}{-2 \cdot \ln 3} = \boxed{1}$$

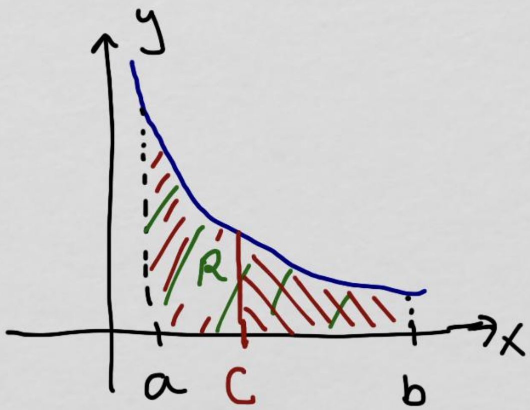
23. $\lim\limits_{x\to 0} \frac{2x-\sin^{-1} x}{2x+\tan^{-1} x} =$ A. 1/2 B. 2 C. 1/3 D. 1 E. 0.

$$\lim_{x\to 0} \frac{2x-\sin^{-1}x}{2x+\tan^{-1}x} \to \frac{0}{0}$$

l'Hospital's Rule only with $\frac{0}{0}$ or $\frac{\infty}{\infty}$

$$\overset{L}{=} \lim_{x\to 0} \frac{2-\frac{1}{\sqrt{1-x^2}}}{2+\frac{1}{1+x^2}} = \frac{2-1}{2+1} = \boxed{\frac{1}{3}}$$

25. Let R be the region between the graph of $y = \frac{1}{x}$ and the x-axis, from $x = a$ to $x = b$ $(0 < a < b)$. If the vertical line $x = c$ cuts R into two parts of equal area, then $c =$ A. $\sqrt{ab}$ B. $\frac{a+b}{2}$ C. $\frac{\ln a + \ln b}{2}$ D. $\ln\left(\frac{a+b}{2}\right)$ E. $\ln\left(\frac{b-a}{2}\right)$



the line $x=c$ cuts R into two equal halves

$$\underbrace{\int_a^c \frac{1}{x}\,dx}_{} = \underbrace{\int_c^b \frac{1}{x}\,dx}_{} = \underbrace{\frac{1}{2}\int_a^b \frac{1}{x}\,dx}_{\text{area of } R}$$

are equal

and combine to form R

$$\int_a^c \frac{1}{x}\,dx = \ln|x|\Big|_a^c = \ln c - \ln a$$

this is equal to

$$\frac{1}{2}\int_a^b \frac{1}{x}\,dx = \frac{1}{2}\ln|x|\Big|_a^b = \frac{1}{2}\ln b - \frac{1}{2}\ln a$$

$$\ln c - \ln a = \frac{1}{2}\ln b - \frac{1}{2}\ln a$$

$$\ln C = \frac{1}{2}\ln b + \frac{1}{2}\ln a$$

$$= \ln b^{1/2} + \ln a^{1/2}$$

$$= \ln\left(a^{1/2} b^{1/2}\right)$$

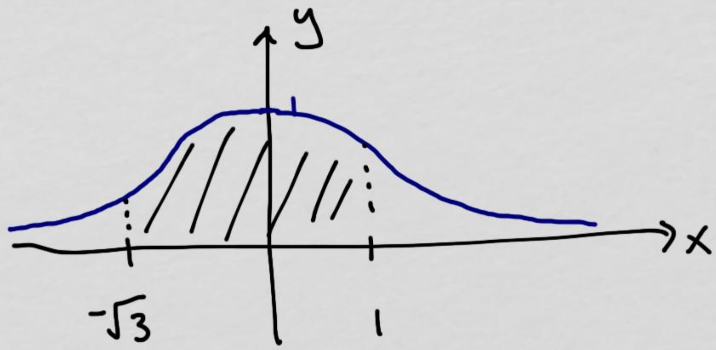
because $\ln xy = \ln x + \ln y$

$$\ln c = \ln \sqrt{ab}$$

so, $\boxed{C = \sqrt{ab}}$

26. Find the area of the region between the graph of $y = \frac{1}{1+x^2}$ and the x–axis, from $x = -\sqrt{3}$ to $x = 1$.

A. $\frac{\pi}{2}$ B. $\frac{3\pi}{4}$ C. $\frac{15\pi}{12}$ D. $\frac{\pi}{3}$ E. $\frac{7\pi}{12}$



$$\int_{-\sqrt{3}}^{1} \frac{1}{1+x^2}\,dx$$

antideriv. is $\tan^{-1} x$

$$= \tan^{-1} x \Big|_{-\sqrt{3}}^{1}$$

$$= \underbrace{\tan^{-1}(1)}_{QI} - \underbrace{\tan^{-1}(-\sqrt{3})}_{QIV}$$

range of $\tan^{-1} x$ is $(-\pi/2, \pi/2)$

$\tan^{-1}\left(\frac{-\sqrt{3}/2}{1/2}\right)$

$$= \frac{\pi}{4} - \left(-\frac{\pi}{3}\right) = \frac{\pi}{4} + \frac{\pi}{3} = \boxed{\frac{7\pi}{12}}$$

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7. Find the slope of the tangent line to the curve

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$$\ln(xy) = x^2 - y^2$$

implicit differentiation

$\underbrace{\ln x + \ln y}$

deriv. does not require product rule

at $(x, y) = (1, 1)$.

A. $\frac{1}{3}$

B. 0

C. -1

D. $\frac{2}{3}$

E. 1

$$\frac{d}{dx} \ln(xy) = \frac{d}{dx}(x^2 - y^2)$$

$$\frac{1}{xy} \frac{d}{dx}(xy) = 2x - 2y\frac{dy}{dx}$$

prod. rule

$$\frac{1}{xy}\left(x\frac{dy}{dx} + y \cdot 1\right) = 2x - 2y\frac{dy}{dx}$$

now plug in $(x, y) = (1, 1)$ only AFTER derivative

$$\frac{dy}{dx} + 1 = 2 - 2\frac{dy}{dx}$$

$$3\frac{dy}{dx} = 1 \qquad \frac{dy}{dx} = \boxed{\frac{1}{3}}$$

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19. Suppose that $\int_1^8 f(x)\,dx = 4$, $\int_5^8 f(x)\,dx = 7$, and $\int_5^1 g(x)\,dx = 3$. Find

$$\boxed{\int_1^5 (f(x) - 2g(x))\,dx.}$$

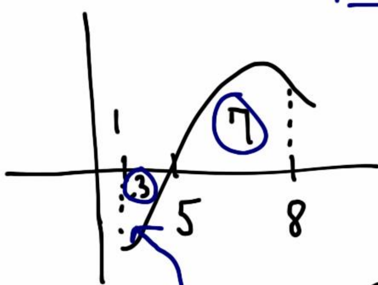
A. -9

B. 0

C. 1

D. 3

E. 5



lower limit is greater than upper limit

rewrite: $-\int_1^5 g(x)\,dx = 3$

$$\underbrace{\int_1^5 f(x)\,dx}_{-3} - \underbrace{\int_1^5 2g(x)\,dx}$$

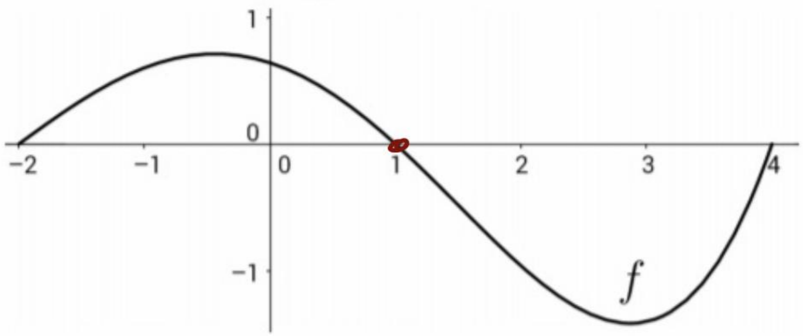
if $-\int_1^5 g(x)\,dx = 3$

$\int_1^5 g(x)\,dx = -3$

$$= \int_1^5 f(x)\,dx - 2\boxed{\int_1^5 g(x)\,dx}$$

$$= -3 - 2(-3) = -3 + 6 = \boxed{3}$$

20. Suppose $A(x) = \int_0^x f(t)\,dt$ where the graph of f is pictured below.



At what x value does $A(x)$ attain its maximum on the interval $-2 \leq x \leq 4$?

A. $x = -2$

B. $x = 0$

C. $x = 1$

D. $x = 3$

E. $x = 4$

$A' = 0$ at some point

and $A' > 0$ to the left and $A' < 0$ to the right

$A(x) = \int_0^x f(t)\,dt \rightarrow A'(x) = \frac{d}{dx}\int_0^x f(t)\,dt = f(x)$

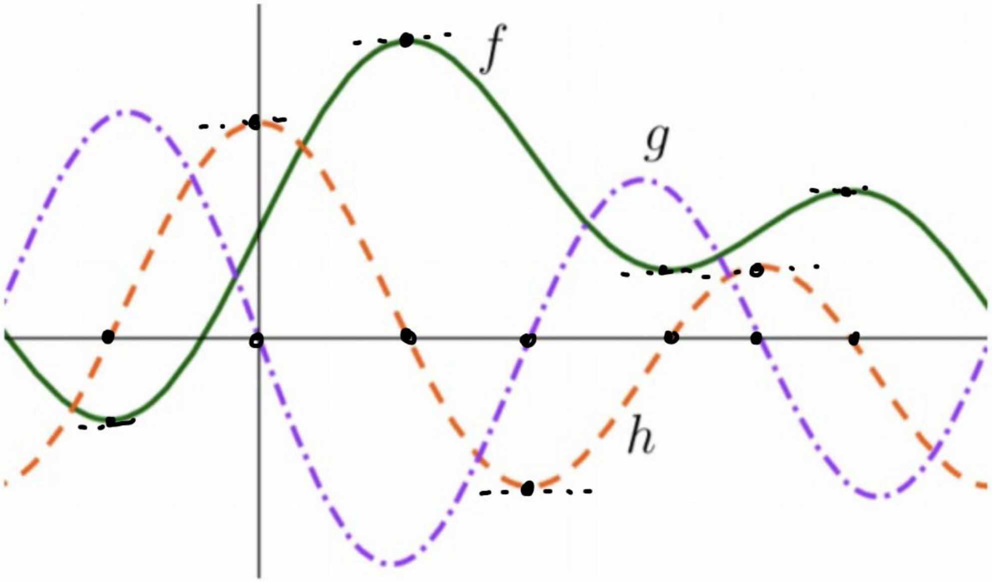
by FTC 1

$A' = 0 \rightarrow f(x) = 0 \rightarrow$ at $x = 1$

and if $x < 1$, $f > 0$ so $A' > 0$, $x > 1$, $f' < 0$ so $A' < 0$

so, A must have a maximum at $x = 1$

25. The graph of a function, its derivative, and one of its antiderivatives is pictured below.



when h has horiz. tangent, $g = 0$, which means $\boxed{h' = g}$

h is an antiderivative of g
or
g is a derivative of h

when f has horiz. tangent, $h = 0$, so $\boxed{f' = h}$

h is a derivative of f
or
f is an antiderivative of h

A. f is an antiderivative of g and h is the derivative of g.

B. h is an antiderivative of g and f is the derivative of g.

C. g is an antiderivative of f and h is the derivative of f.

D. h is an antiderivative of f and g is the derivative of f.

E. f is an antiderivative of h and g is the derivative of h.