4.4 Graphing Functions (part 1)

Sketching Guidelines

Domain

Symmetry $\longrightarrow$ not useful, ok skip

Increasing / Decreasing Intervals
Relative Max/min
} deal w/ f'

Concave up/down Intervals
Inflections
} deal w/ f''

Asymptotes

Intercepts

Graph

example

$f(x) = x^3 - 12x^2 + 36x$

Domain: $(-\infty, \infty)$ because $f(x)$ is polynomial

Symmetry: skip

Inc/Dec intervals: $f'(x) = 3x^2 - 24x + 36$

$= 3(x^2 - 8x + 12)$

$= 3(x-6)(x-2)$

$f' = 0 \rightarrow x = 6,\ x = 2$

f' DNE $\rightarrow$ never

f': $+ +$ 0 $- -$ 0 $+ +$

$\nearrow$ $\searrow$ $\nearrow$

$x=2$ $x=6$

inc: $(-\infty, 2), (6, \infty)$

dec: $(2, 6)$

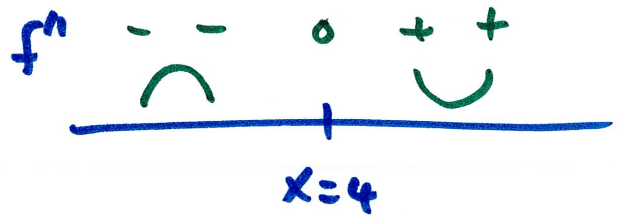
Relative Max/Min: rel. max at $x=2$, $y = f(2) = 32 \rightarrow (2, 32)$

rel. min at $x=6$, $y = f(6) = 0 \rightarrow (6, 0)$

CU/CD intervals: $f'' = 6x - 24$

$f'' = 0 \rightarrow \quad x = 4$

f'' DNE $\rightarrow$ never



CU: $(4, \infty)$

CD: $(-\infty, 4)$

Inflection pts: at $x = 4$, $y = f(4) = 16 \rightarrow (4, 16)$

Asymptotes: none. $f(x)$ is polynomial

Intercepts: x-intercept @ $y = 0$

$f(x) = x^3 - 12x^2 + 36x$

$0 = x^3 - 12x^2 + 36x$

$0 = x(x^2 - 12x + 36)$

$0 = x(x-6)(x-6)$

$x = 0, \quad x = 6$

y-int @ $x = 0$ $\quad y = 0$

points we know: x-ints: $x = 0$, $x = 6$

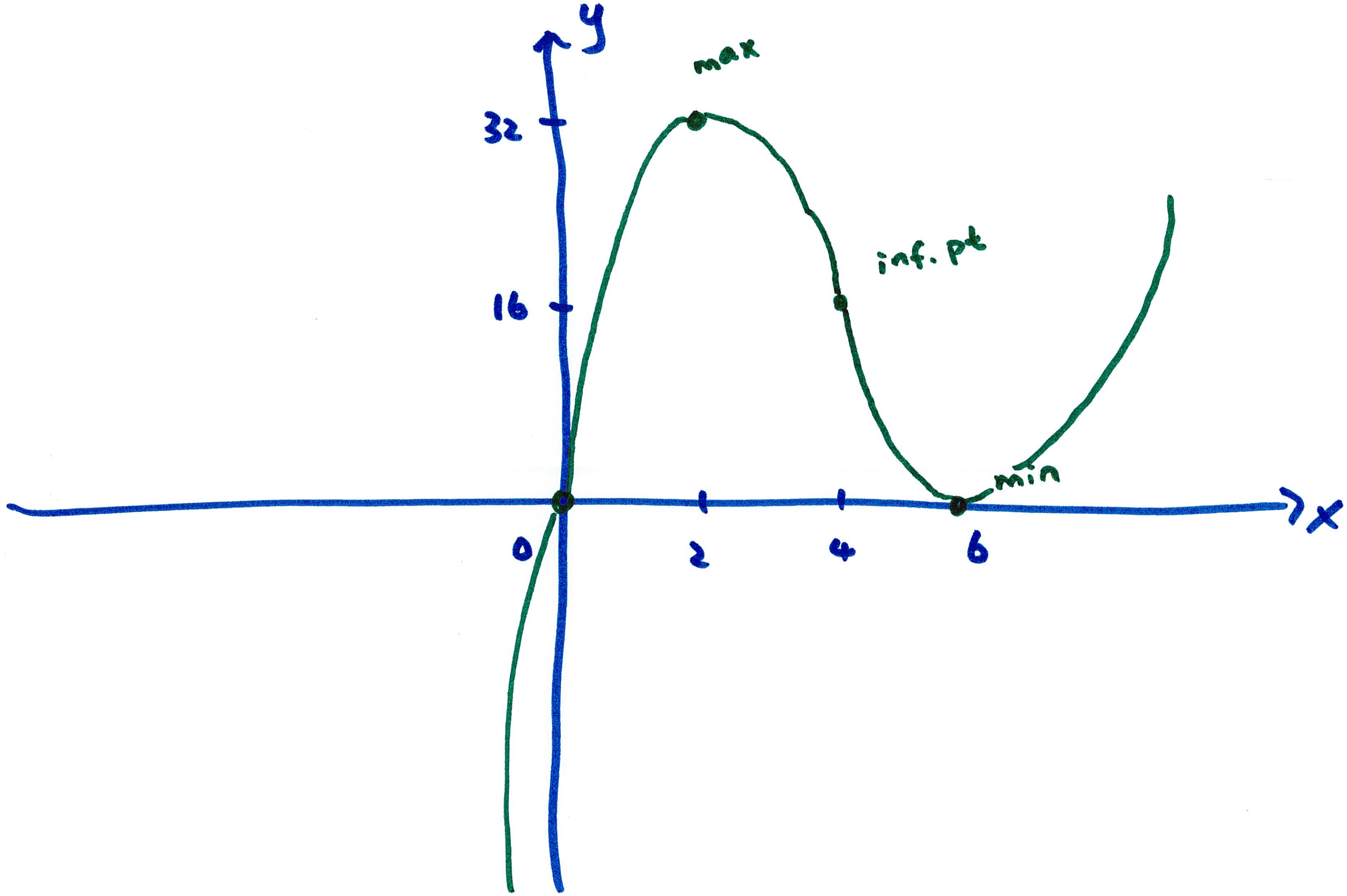
y-ints: $y = 0$

rel. max: $(2, 32)$

rel. min: $(6, 0)$

inf. pt: $(4, 16)$

start w/ these, then fill in details w/ CU/CD or inc/dec info



CU: $(4, \infty)$

CD: $(-\infty, 4)$

example $f(x) = \dfrac{x}{x^2-16}$

Domain: $x \neq 4$, $x \neq -4$ → these are vertical asymptotes (denom $= 0$ numer $\neq 0$)

Inc/Dec: $\cancel{f' = - \dfrac{x^2+16}{(x^2-4}}$

$$f' = -\frac{x^2+16}{(x^2-16)^2}$$

$f' = 0 \rightarrow x^2+16 = 0 \rightarrow$ no solutions

f' DNE $\rightarrow x^2-16=0 \rightarrow x=4,\ x=-4$

f': − − DNE − − DNE − −

↘ ↘ ↘

$x=-4$ $x=4$

dec: $(-\infty,-4)$, $(-4,4)$, $(4,\infty)$

inc: none

Rel. max/min: none, no f' sign change

besides, $x = \pm 4$ couldn't be locations of max/min because vertical asymptotes

CU/CD: $f''(x) = \dfrac{2x(x^2+48)}{(x^2-16)^3}$

$f'' = 0 \rightarrow 2x(x^2+48) = 0 \rightarrow x = 0$

($x^2+48=0$ has no solutions)

f'' DNE $\rightarrow x = \pm 4$

	$(-\infty,-4)$	$x=-4$	$(-4,0)$	$x=0$	$(0,4)$	$x=4$	$(4,\infty)$
f''	$- \; -$	DNE	$+ \; +$	0	$- \; -$	DNE	$+ \; +$
	$\cap$		$\cup$		$\cap$		$\cup$

CU: $(-4, 0), (4, \infty)$

CD: $(-\infty, -4), (0, 4)$

Inf. pts: at $x=0$ only because even though there are sign changes at $x = \pm 4$, ~~th~~ $x = \pm 4$ are not in domain, so no points exist there

inf. pt: $x = 0$, $y = f(0) = 0 \rightarrow (0, 0)$

Asymptotes: vertical : $x = 4$, $x = -4$

horizontal: $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$

$$\left.\begin{array}{l} \lim_{x\to\infty} \dfrac{x}{x^2-16} = 0 \\[2ex] \lim_{x\to-\infty} \dfrac{x}{x^2-16} = 0 \end{array}\right\} \quad y = 0 \text{ horiz. asymp.}$$

Intercepts: x-int : @ $y=0$

$$f(x) = \frac{x}{x^2-16}$$

$$0 = \frac{x}{x^2-16} \rightarrow x = 0$$

y-int : @ $x=0$

$$f(x) = \frac{x}{x^2-16}$$

$$y = 0 \rightarrow y = 0$$

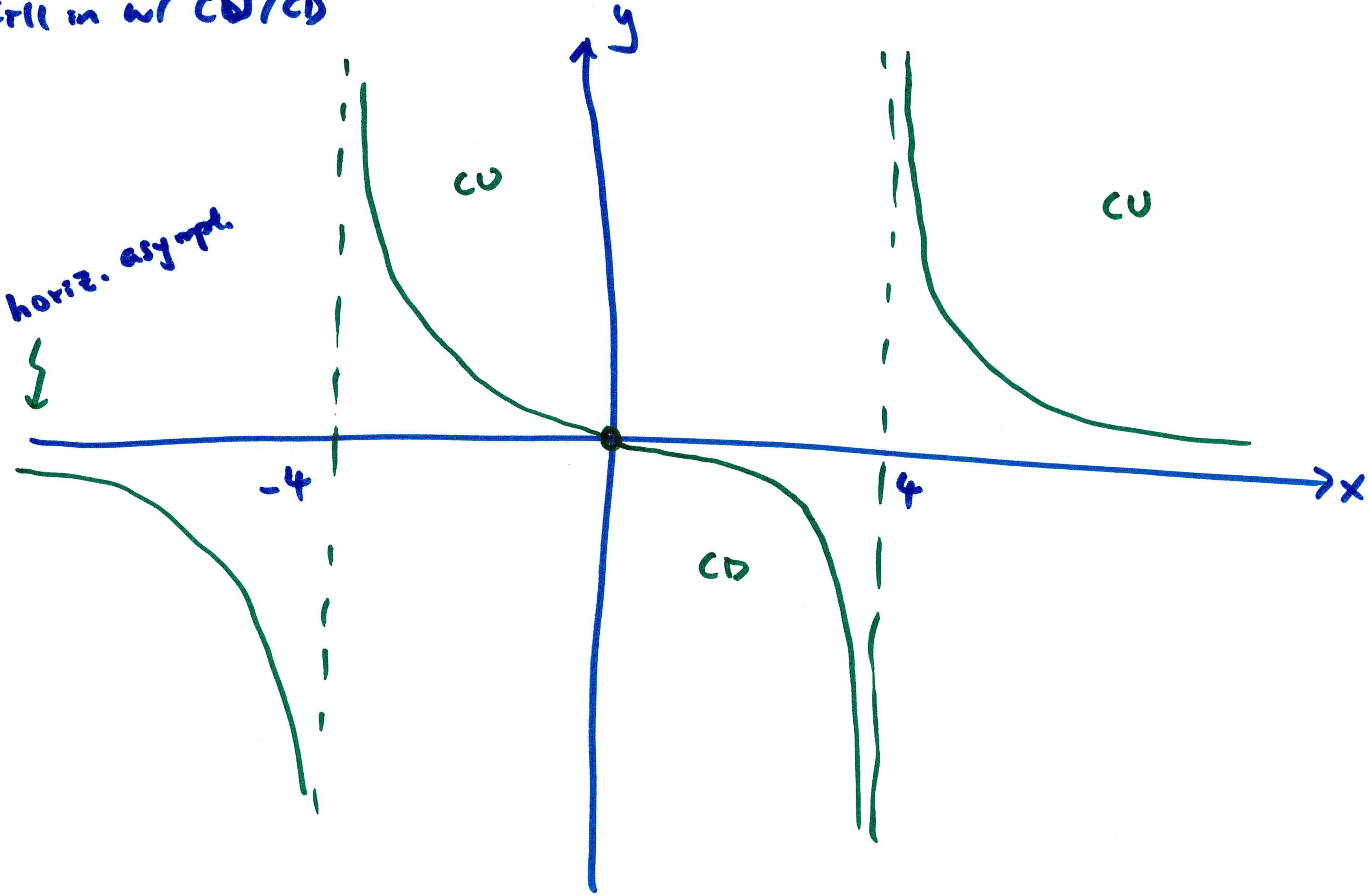
points we know:

x-int: $x=0$
y-int: $y=0$
inf. pt: $(0,0)$

all same point

CU: $(-4, 0)$, $(4, \infty)$
CD: $(-\infty, -4)$, $(0, 4)$

then asympt
then fill in w/ CU/CD



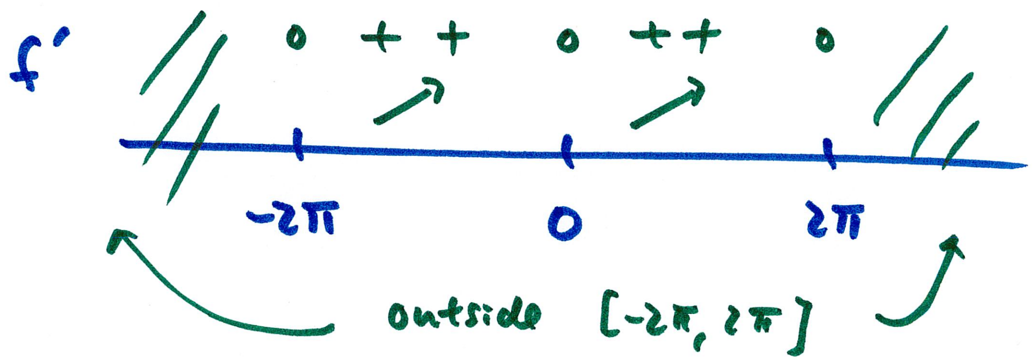
<u>example</u> $f(x) = x - \sin x$ on $[-2\pi, 2\pi]$

Domain: $[-2\pi, 2\pi]$

Inc/Dec: $f'(x) = 1 - \cos x$

$f' = 0 \rightarrow \cos x = 1 \rightarrow x = -2\pi,\ x = 0,\ x = 2\pi$

f' DNE $\rightarrow$ never

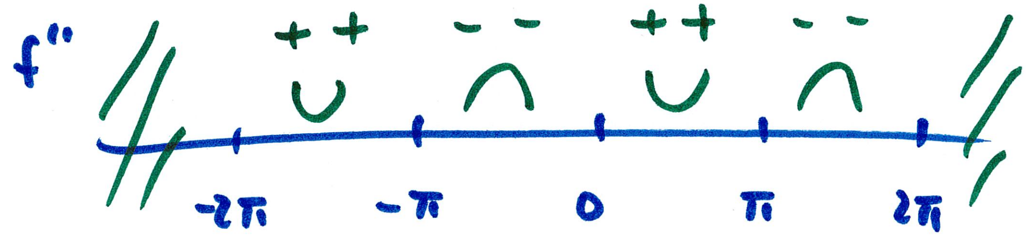


Rel. max/min: none (no f' sign change)

CU/CD: $f''(x) = \sin x$

$f'' = 0 \rightarrow \sin x = 0 \rightarrow x = -2\pi,\ x = -\pi,\ x = 0,\ x = \pi,\ x = 2\pi$

f'' DNE $\rightarrow$ never



Inf. pts : $x = -\pi$, $y = f(-\pi) = -\pi \rightarrow (-\pi, -\pi)$

$x = 0$, $y = f(0) = 0 \rightarrow (0, 0)$

$x = \pi$, $y = f(\pi) = \pi \rightarrow (\pi, \pi)$

Asymptotes : none

Intercepts: x-int: @ $y=0$

$$f(x) = x - \sin x$$
$$0 = x - \sin x$$
$$x = \sin x \rightarrow x = 0$$

y-int: @ $x=0$

$$f(x) = x - \sin x$$
$$y = 0 - \sin 0 = 0 \rightarrow y = 0$$

points we know: inf. pts: $(-\pi, \pi)$, $(0, 0)$, (π, π)

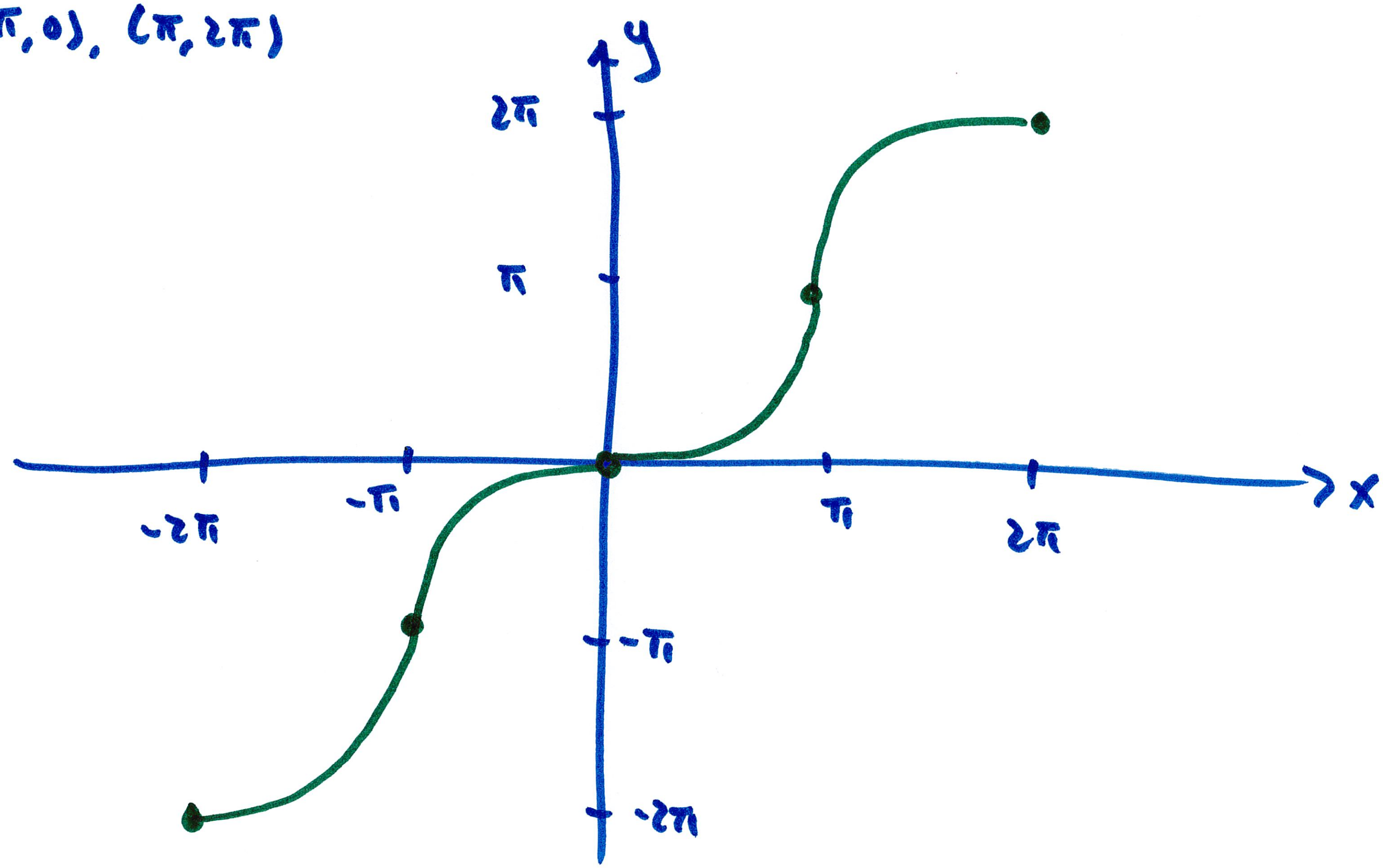
intercepts: $(0, 0)$

endpoints: $x = -2\pi$, $y = f(-2\pi) \to (-2\pi, -2\pi)$

$x = 2\pi$, $y = f(2\pi) \to (2\pi, 2\pi)$

CU: $(-2\pi, -\pi)$, $(0, \pi)$

CD: $(-\pi, 0)$, $(\pi, 2\pi)$



Some functions have slant asymptotes

$$f(x) = \frac{x^2+15}{5x+1}$$

notice as $x \to \infty$, $x^2+15 \approx x^2$, $5x+1 \approx 5x$

so, $\dfrac{x^2+15}{5x+1} \to \dfrac{x^2}{5x} \to \dfrac{1}{5}x$

this means the graph will approach $y = \frac{1}{5}x$

as $x \to \pm\infty$

vertical : $x = -\frac{1}{5}$

go through CU/CD step

CU: $(-\frac{1}{5}, \infty)$

CD: $(-\infty, -\frac{1}{5})$