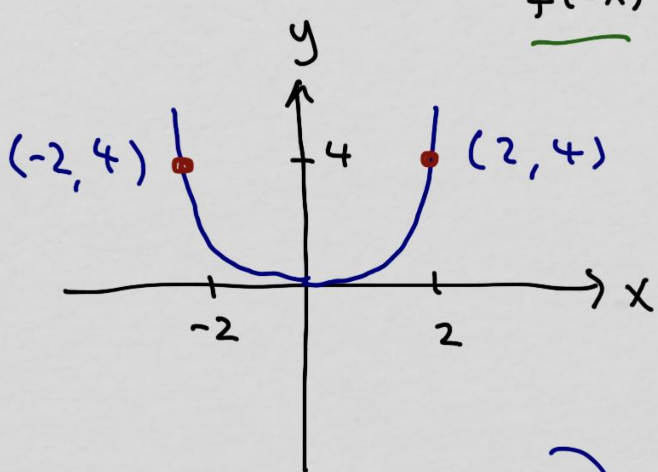
# 4.4 Graphing Functions (part 2)

Symmetry : y-axis , origin

y-axis sym: $f(x)$ is <u>even</u> $\rightarrow$ $f(-x) = f(x)$

example: $f(x) = x^2$

$f(-x) = (-x)^2 = x^2 = f(x)$

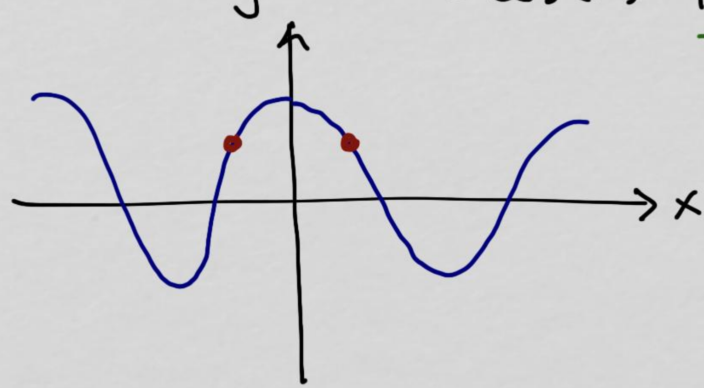



another example

$f(x) = \cos(x)$

$f(-x) = \cos(-x)$

$= \cos(x) = f(x)$



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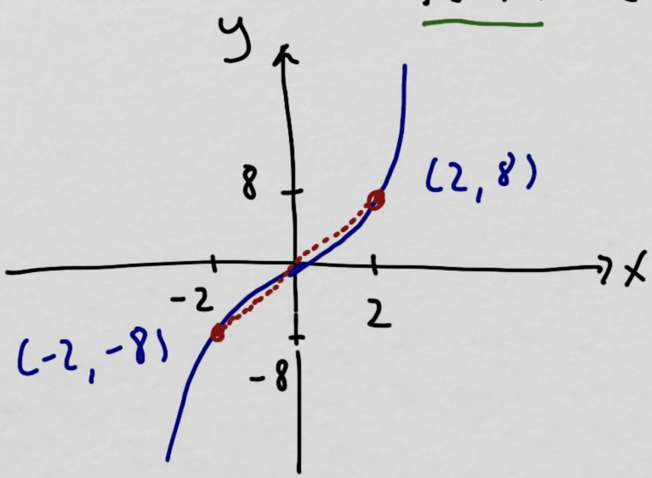
origin sym: $f(x)$ is odd $\rightarrow f(-x) = -f(x)$

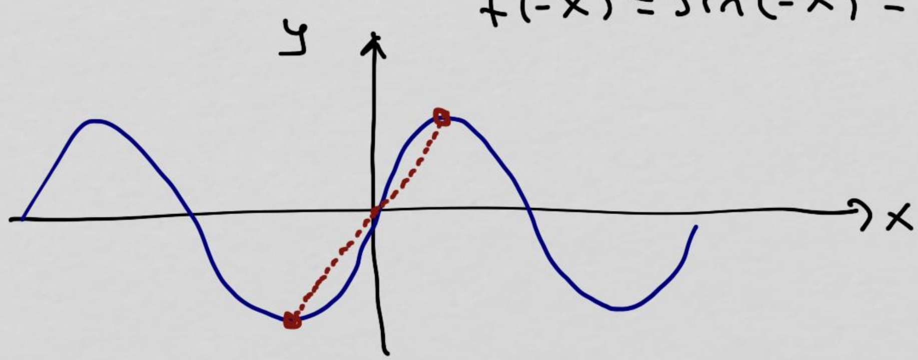
example: $f(x) = x^3$

$f(-x) = (-x)^3 = -x^3 = -f(x)$




another example: $f(x) = \sin(x)$

$f(-x) = \sin(-x) = -\sin(x) = -f(x)$



The first derivative alone is enough to get a reasonable sketch

Example Sketch $f(x)$ if we know $f'(x) = (x-2)(x+1)(x+5)$

find critical #s: $f' = 0 \rightarrow \boxed{x = 2, -1, -5}$ potential locations of rel. max/min need to have $f'$ changing sign

$f'$ DNE $\rightarrow$ never

$f'$ $-\ -$ $0$ $+\ +$ $0$ $-\ -$ $0$ $+\ +$

$\searrow$ $\nearrow$ $\searrow$ $\nearrow$

$x = -5$ $x = -1$ $x = 2$

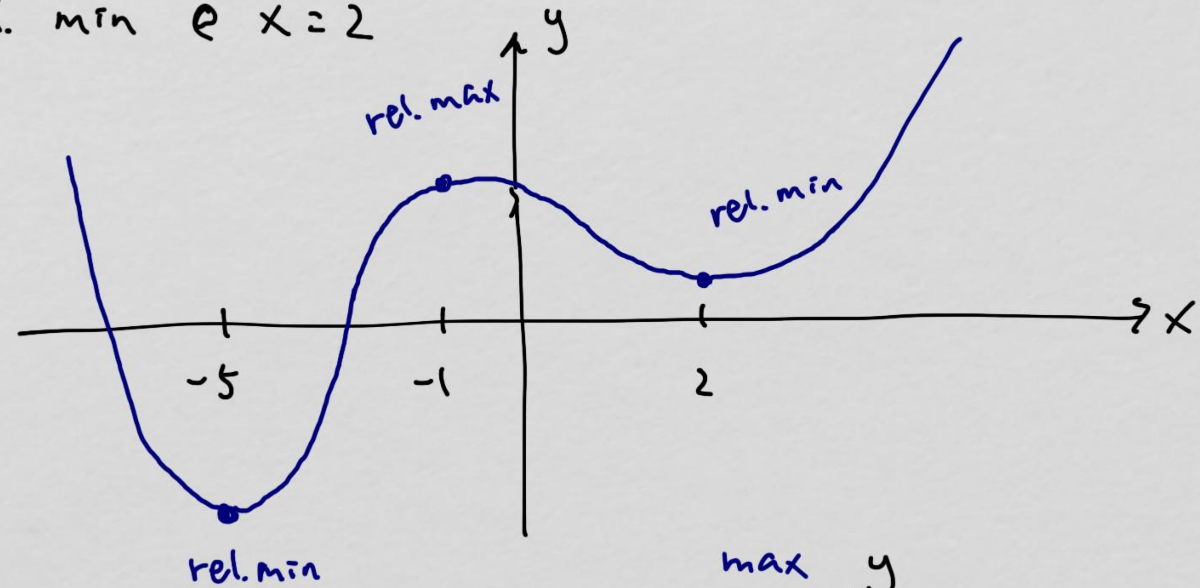
rel. min @ $x = -5$
rel. max @ $x = -1$
rel min @ $x = 2$

$\Big\}$ $y = ?$ we don't know $f(x)$

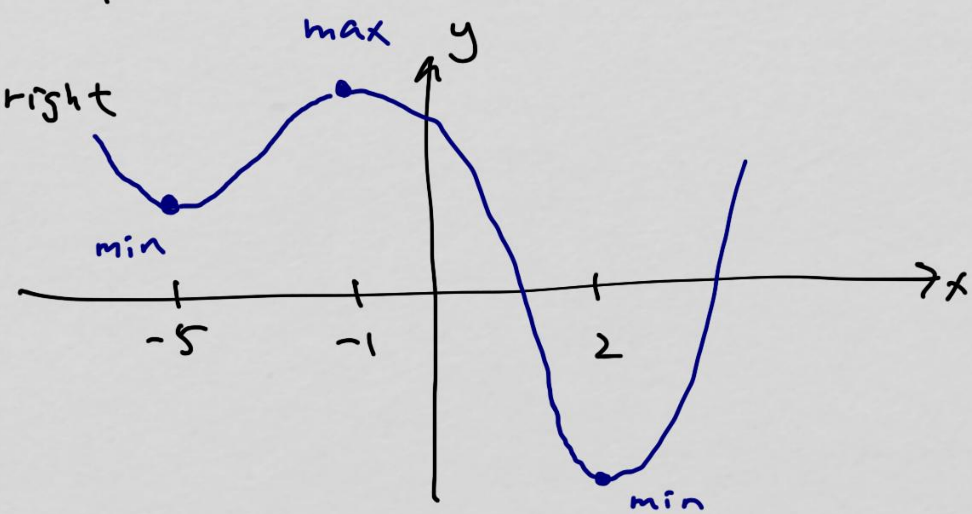
So, when we sketch, make up a $y$ for each of them

$\rightarrow$ we get accurate inc/dec behavior but not the exact location of points rel. min @ $x = -5$ rel. max @ $x = -1$

rel. min @ $x = 2$




this is equally right




graph based on
first deriv. alone
is NOT unique

example

$f'(x) = \sin 3x$ on $\left[-\frac{4\pi}{3}, \frac{4\pi}{3}\right]$

$-\frac{4\pi}{3} \le x \le \frac{4\pi}{3}$

$\downarrow$

$-4\pi \le x \le 4\pi$

use this to solve

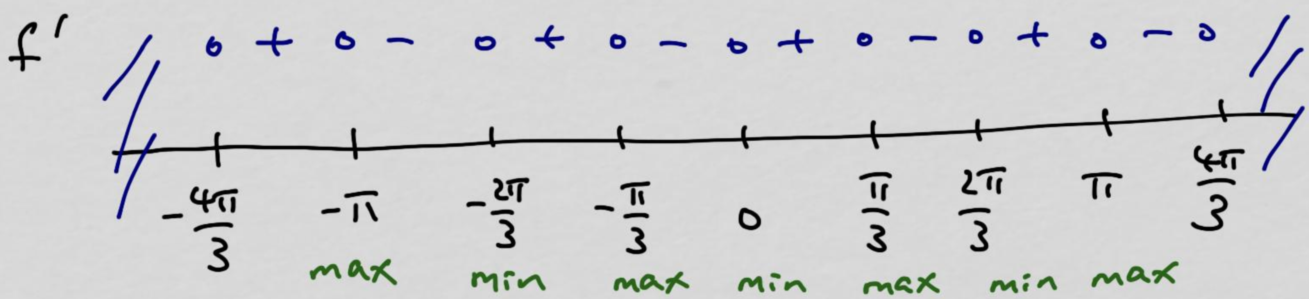
$\sin(3x) = 0$

$f' = 0 \to \underbrace{\sin(3x) = 0}_{\text{solution is } 3x = \dots}$

$\sin(3x) = 0$

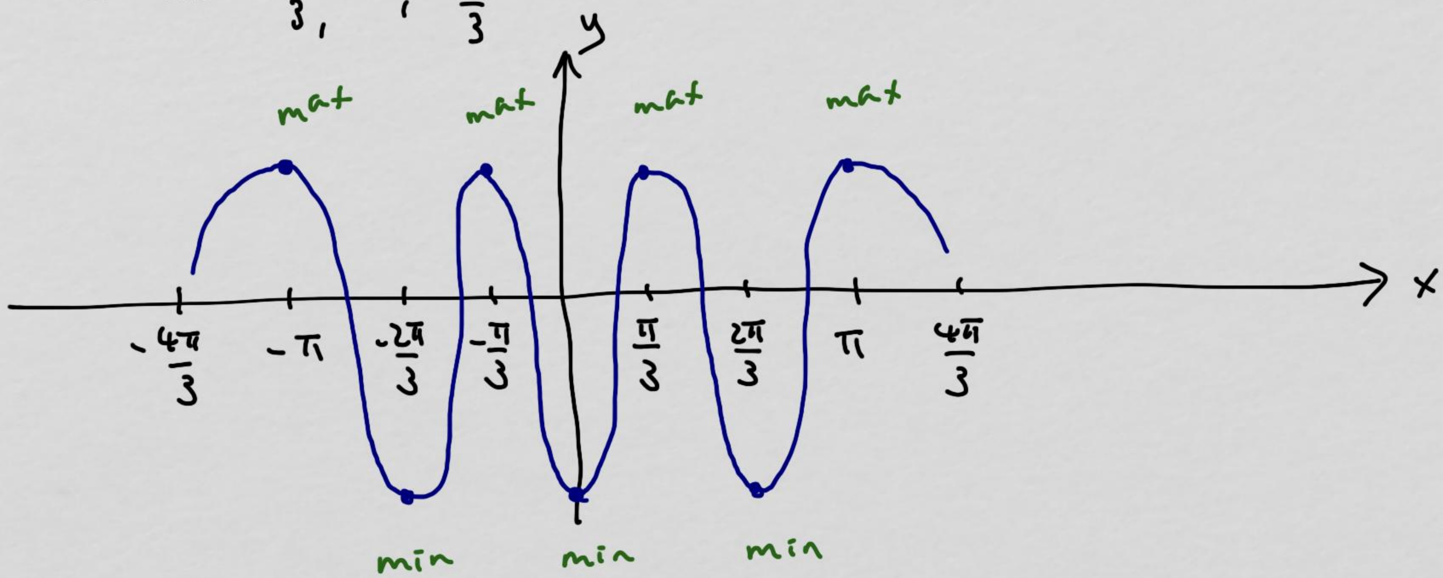
$3x = -4\pi, -3\pi, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi, 4\pi$

$x = -\frac{4\pi}{3}, -\pi, -\frac{2\pi}{3}, -\frac{\pi}{3}, 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$



rel. max @ $x = -\pi, -\frac{\pi}{3}, \frac{\pi}{3}, \pi$

rel. min @ $x = -\frac{2\pi}{3}, 0, \frac{2\pi}{3}$



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<u>example</u> Sketch $f(x) = x^x$ using the first derivative

$y = x^x$ log differentiation to find $y'$

$\ln y = \ln x^x$

$= (x)(\ln x)$ differentiate implicitly

$\frac{1}{y} y' = x \cdot \frac{1}{x} + \ln x \cdot 1$ prod. rule

$= 1 + \ln x$

$y' = y(1 + \ln x)$

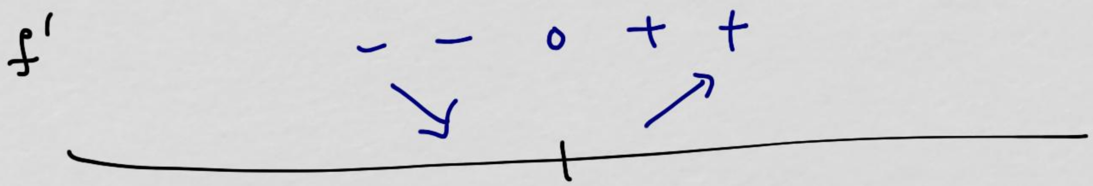
$f'(x) = x^x (1 + \ln x)$

$f' = 0 \rightarrow x^x = 0 \rightarrow$ never $(0^0 = 1)$

$1 + \ln x = 0$

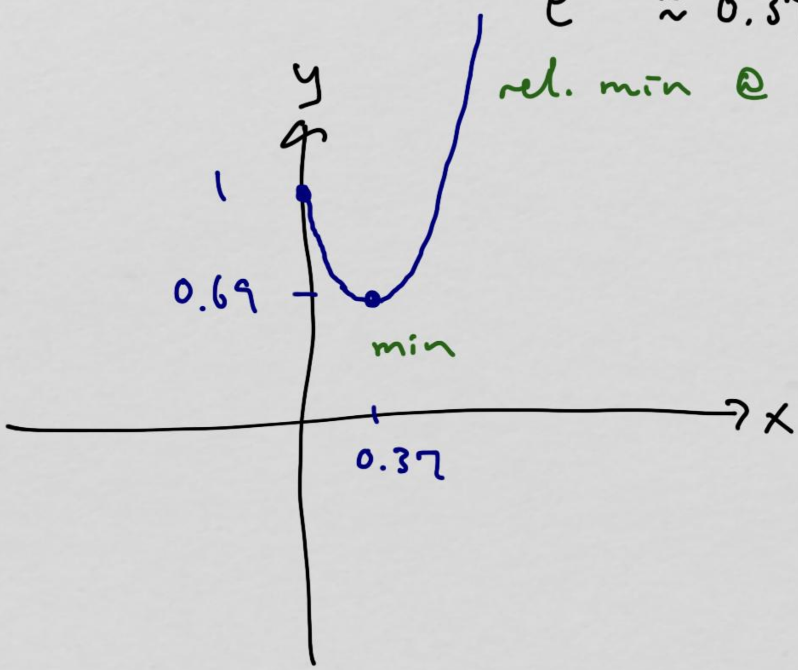
$\ln x = -1 \rightarrow x = e^{-1}$

$$f'(x) = x^x(1+\ln x)$$




$e^{-1} \approx 0.37$

rel. min @ $x = e^{-1}$ $\quad y = x^x = (e^{-1})^{e^{-1}} \approx 0.69$




nothing for $x<0$

why?

$$a^b = e^{\ln a^b} = e^{b \ln a}$$

so, $x^x = e^{x \ln x}$

this is why nothing for $x<0$