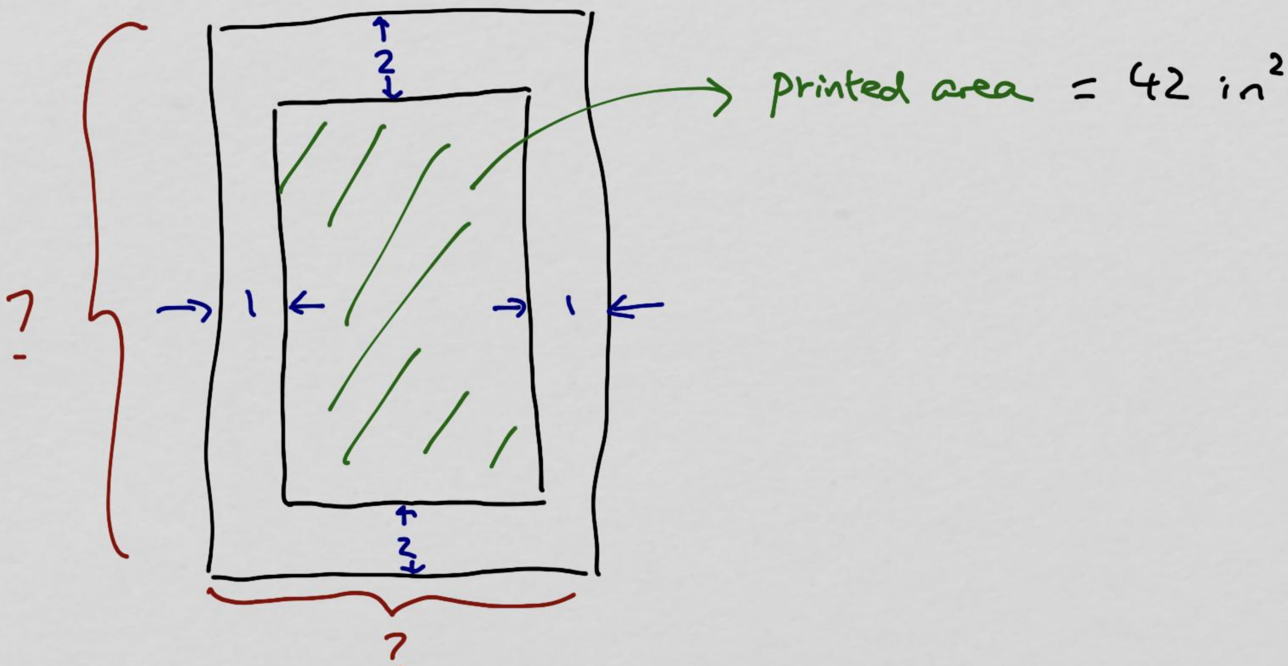
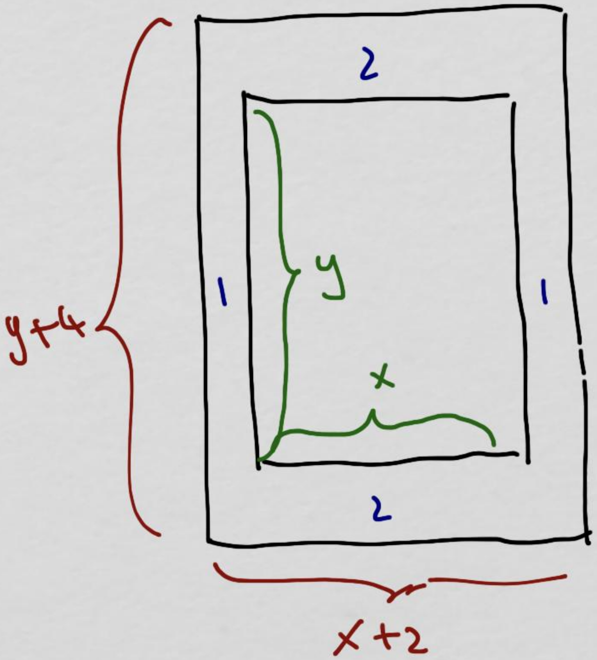
4.5 Optimization Problems (part 2)

example A page from a book is to have a printed area of $42\ in^2$, and the margins at the top and bottom are 2 in and the margins at the left and right are 1 in. Find the dimensions of the page to minimize the total area.





x: width of printed area

y: length of printed area

$\boxed{xy = 42}$ (printed area $= 42 \text{ in}^2$)

width of the entire page: $x+1+1 = x+2$

length of the entire page: $y+2+2 = y+4$

objective: minimize $\boxed{A = (x+2)(y+4)}$

$$y = \frac{42}{x}$$

$$A = (x+2)\left(\frac{42}{x}+4\right)$$

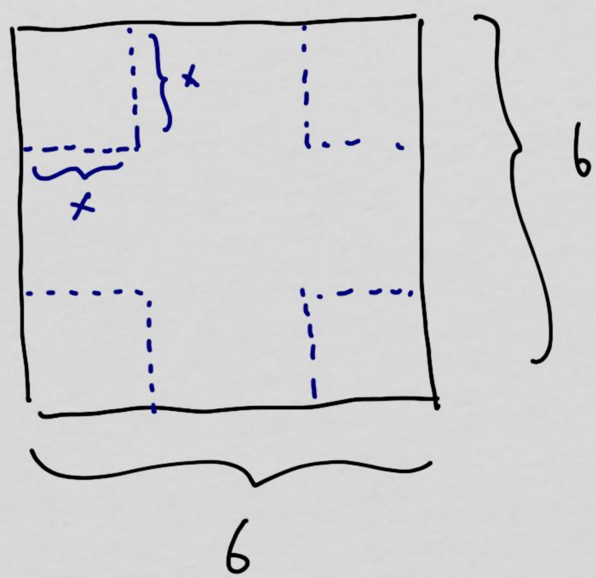
$0 \leq x < \infty$
(not bounded)

take derivative, find critical #s, verify max/min.

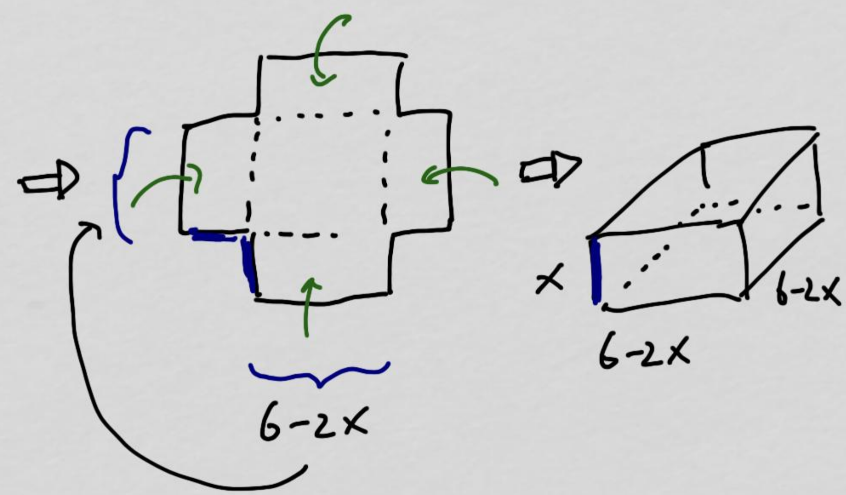
<u>example</u> A piece of cardboard is 6 in by 6 in

Small squares are removed from the corners and then fold the flaps to make a box.

Find the volume of the largest possible box.



X: side of each small square



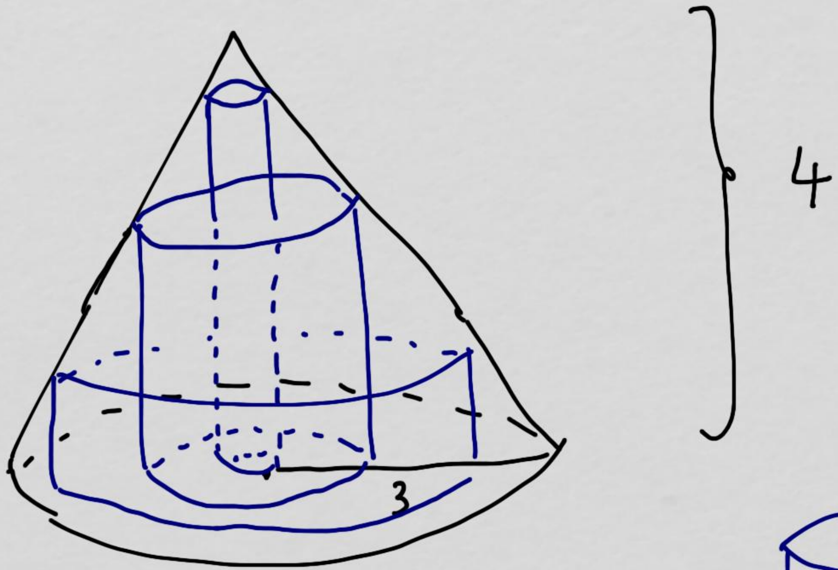
box volume : $V(x) = (6-2x)^2(x)$ $\qquad 0 \le x \le 3$

take deriv, find critical #s then compare V at endpts and critical numbers.

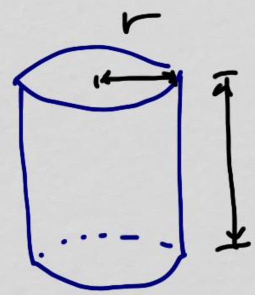
no squares take from corners

each square is 1/4 of cardboard

example Find the dimensions of the right circular cylinder of the largest volume that can be inscribed in a cone of height 4 and radius 3.

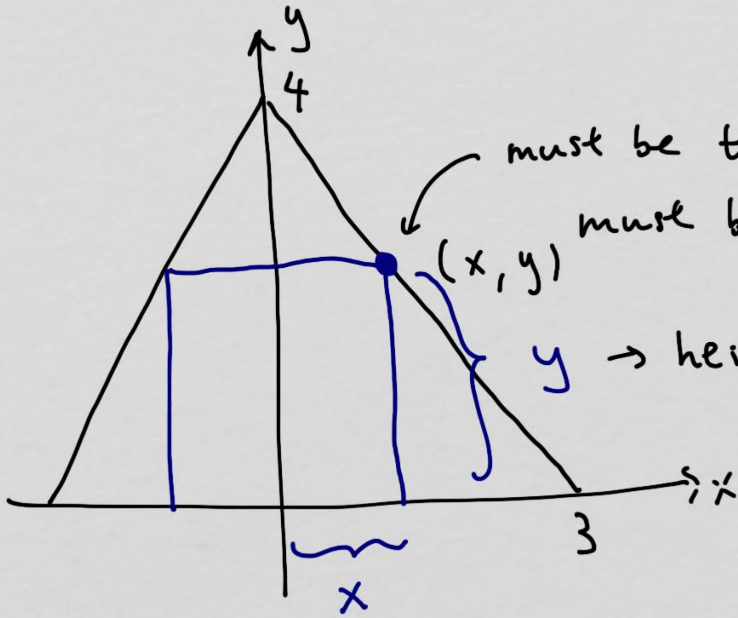


largest possible cylinder?



volume of cylinder is $\pi r^2 h$

slice through the cone and cylinder and put on xy axes



must be touching cone

must be on the line through (3, 0) and (0, 4)

$y \rightarrow$ height of cylinder

↳ radius of cylinder

volume of cylinder: $\pi x^2 y = V$

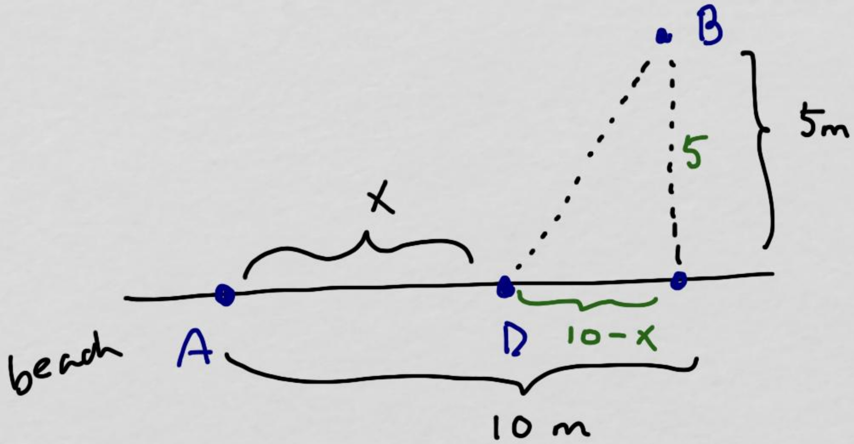
eg. of line through $(0, 4)$ and $(3, 0)$ $\rightarrow$ $\boxed{y = -\frac{4}{3}x + 4}$

use this to remove y from V

$$\boxed{V(x) = \pi x^2 \left(-\frac{4}{3}x + 4\right) \qquad 0 \le x \le 3}$$

find critical pts, compare V at ends and critical pts.

<u>example</u> A dog can run at 5 m/s and swim at 1 m/s.



Dog at A wants to get to ball at B.

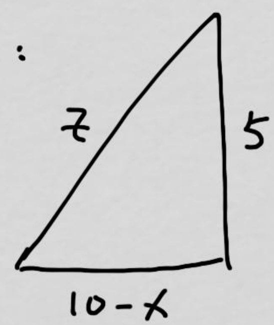
Dog runs along beach to D then jumps in water and swims the rest of the way.

Where should D be to minimize the time to reach the ball?

x : distance to run on land

time spent on land : $T_L = \dfrac{\text{dist.}}{\text{speed}} = \dfrac{x}{5}$

water portion :



distance to swim :

$$z = \sqrt{5^2 + (10-x)^2}$$

Draw Erase Select Point Add Undo Redo Rec Stop View Close

time spent in water : $T_w = \dfrac{\text{dist. in water}}{\text{speed in water}}$

$$T_w = \frac{\sqrt{25+(10-x)^2}}{1} = \left(x^2-20x+125\right)^{1/2}$$

total time : $\boxed{T = \underbrace{\frac{x}{5}}_{\text{land}} + \underbrace{\left(x^2-20x+125\right)^{1/2}}_{\text{water}} \qquad 0 \le x \le 10}$

jump in immediately swim the entire way

run until even w/ the ball then swim

find critical #'s, compare T at ends and at critical #s

exam 3 covers up to this lesson.