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5.5 Substitution Rule (part 2)

more complicated examples

general principle : u = part of integral whose derivative is some constant multiple of another part

example $\displaystyle\int \sin^3(2x)\cos(2x)\,dx$

$$= \int [\sin(2x)]^3 \cos(2x)\,dx$$

$$\frac{d}{dx}\sin(2x) = \cos(2x)\cdot 2 = 2\cos(2x) \rightarrow \text{2 times the other part}$$

so, let $\boxed{u = \sin(2x)}$

then $\dfrac{du}{dx} = \cos(2x)\cdot 2$

$$\boxed{du = 2\cos(2x)\,dx}$$

sub u, du into original integral

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$$\int \underbrace{[\sin(2x)]^3}_{u^3} \underbrace{\cos(2x)\,dx}_{\frac{1}{2}du}$$

$u = \sin(2x)$

$du = 2\cos(2x)\,dx$

$\frac{1}{2}du = \cos(2x)\,dx$

$$= \int \frac{1}{2}u^3\,du = \frac{1}{2}\int u^3\,du = \frac{1}{2}\left(\frac{u^4}{4}\right) + C = \frac{1}{8}(\sin(2x))^4 + C$$

$$= \boxed{\frac{1}{8}\sin^4(2x) + C}$$

what if we chose the wrong u?

$$\int [\sin(2x)]^3 \cdot \cos(2x)\,dx$$

if we let $u = [\sin(2x)]^3$ $\quad \frac{du}{dx} = 3[\sin(2x)]^2 \cdot \cos(2x) \cdot 2$

$$du = \underbrace{6\sin^2(2x)}_{\text{we don't have this}} \boxed{\cos(2x)\,dx}$$

this is all we have

we need the entire thing to form du

we can't form du, so we can't proceed.

another possible wrong u

$$\int [\sin(2x)]^3 \cdot \cos(2x)\,dx$$

if $u = \cos(2x)$ $\quad \dfrac{du}{dx} = -\sin(2x)\cdot 2$

$du = -2\sin(2x)\,dx$

$$\int \underbrace{[\sin(2x)]^2}_{?} \cdot \underbrace{\cos(2x)}_{u} \cdot \underbrace{\sin(2x)\,dx}_{-\frac{1}{2}du}$$

how do we get
rid of this?
it must be turned into
something to do with u

If things get too busy or complicated → re-visit choice of u

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<u>example</u>

$$\int \frac{e^{2x}}{e^{2x}+3}\,dx$$

$$= \int (\boxed{e^{2x}+3})^{-1} \cdot \boxed{e^{2x}}\,dx$$

the deriv. of $e^{2x}+3$ is $2e^{2x}$, which is a constant multiple of the other part

so, we choose $\boxed{u = e^{2x}+3}$

then $\frac{du}{dx} = e^{2x}\cdot 2$

$\boxed{du = 2e^{2x}\,dx} \longrightarrow \frac{1}{2}du = e^{2x}dx$

$$\int \underbrace{(e^{2x}+3)^{-1}}_{u^{-1}} \cdot \underbrace{e^{2x}\,dx}_{\frac{1}{2}du} = \frac{1}{2}\int u^{-1}du = \frac{1}{2}\ln|u| + C$$

$$= \boxed{\frac{1}{2}\ln\left|e^{2x}+3\right| + C}$$

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Example

$$\int \frac{\sec^2 x}{\tan^3 x}\,dx$$

$$= \int \left(\boxed{\tan x}\right)^{-3} \boxed{\sec^2 x}\,dx$$

$$\frac{d}{dx}\tan x = \sec^2 x \qquad \text{so,} \quad u = \tan x$$

$$\frac{du}{dx} = \sec^2 x$$

$$du = \sec^2 x\,dx$$

$$\int \underbrace{(\tan x)^{-3}}_{u^{-3}} \underbrace{\sec^2 x\,dx}_{du}$$

$$= \int u^{-3}\,du = \frac{u^{-2}}{-2} + C = -\frac{1}{2}u^{-2} + C = -\frac{1}{2}(\tan x)^{-2} + C$$

$$= \boxed{-\frac{1}{2\tan x} + C}$$

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example

$$\int \sec(x-10)\tan(x-10)\,dx$$

this one is a little different

$$\frac{d}{dx}\sec(x-10) = \sec(x-10)\tan(x-10)$$ → is NOT a constant multiple of the other part ($\tan(x-10)$)

$$\frac{d}{dx}\tan(x-10) = \sec^2(x-10)$$ → NOT a constant multiple of the other part ($\sec(x-10)$) either.

there really isn't a lot of choice left

$u = x-10$

$\frac{du}{dx} = 1$

$du = dx$

$$\int \sec\underbrace{(x-10)}_{u}\tan\underbrace{(x-10)}_{u}\,\underbrace{dx}_{u}$$

$$= \int \sec(u)\tan(u)\,du = \sec(u)+C$$

$$= \boxed{\sec(x-10)+C}$$

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<u>example</u>

$$\int_{e^{36}}^{e^{81}} \frac{1}{x\sqrt{\ln x}}\,dx$$

$$= \int_{e^{36}}^{e^{81}} \left(\boxed{\ln x}\right)^{-1/2} \boxed{\frac{1}{x}}\,dx$$

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

so, $u = \ln x$

$$\frac{du}{dx} = \frac{1}{x}$$

$$du = \frac{1}{x}\,dx$$

this is a definite integral, we must adjust the integration limits

old upper limit: $x = e^{81} \rightarrow u = \ln x = \ln e^{81} = 81$

old lower limit: $x = e^{36} \rightarrow u = \ln x = \ln e^{36} = 36$

new integral: $\displaystyle \int_{36}^{81} u^{-1/2}\,du = \left.\frac{u^{1/2}}{1/2}\right|_{36}^{81} = \frac{(81)^{1/2}}{1/2} - \frac{(36)^{1/2}}{1/2} = 2(9-6)$

$= \boxed{6}$

DO NOT go back to x

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