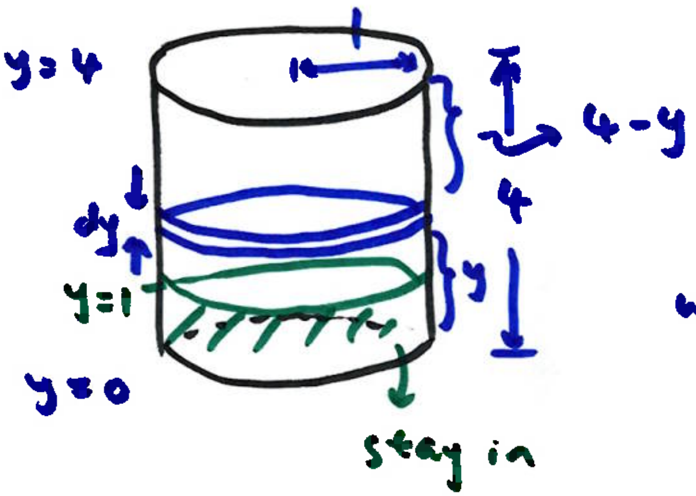
A cylindrical tank of height 4 feet and radius 1 foot is filled with water. How much work is required to pump all but 1 foot of water out of the tank. (Density = 62.5 lbs./ft^3)

A. $9\pi(62.5)$ ft-lbs. B. $3\pi(62.5)$ ft-lbs. C. $\frac{9\pi}{2}(62.5)$ ft-lbs. D. $18\pi(62.5)$ ft-lbs.
E. $6\pi(62.5)$ ft-lbs.

= force, no gravity to multiply

kg is a mass so mult. by $g = 9.8$



work = force · distance

work to move one slice of water then integrate over water to move

work of slice = (weight)(distance to go)

$= (62.5)(\pi)(1)^2(dy)\cdot(4-y)$

$= 62.5\pi(4-y)dy$

$\int_1^4 62.5\pi(4-y)dy$

$= 62.5\pi\left(4y - \frac{1}{2}y^2\right)\Big|_1^4$

$= 62.5\pi\left[(16-8)-\left(4-\frac{1}{2}\right)\right] = 62.5\pi\left(8-\frac{7}{2}\right)$

$= 62.5\pi\left(\frac{9}{2}\right)$

Evaluate $\displaystyle\int_0^1 xe^{3x}\,dx$.

A. $\frac{2e^3}{9}$ B. $\frac{1}{9}+\frac{2e^3}{9}$ C. 1 D. $\frac{1}{9}$ E. $\frac{e^3}{9}-1$

by parts: $uv - \int v\,du$ order to pick u: LIATE

L: log; I: inverse trig; A: algebraic; T: trig; E: exponential

here, x, e^{3x} so $u = x$ $dv = e^{3x}dx$

$x \to$ A, $e^{3x} \to$ E $du = dx$ $v = \frac{1}{3}e^{3x}$

$$uv\Big|_0^1 - \int_0^1 v\,du = \frac{1}{3}xe^{3x}\Big|_0^1 - \int_0^1 \frac{1}{3}e^{3x}dx$$

$$= \frac{1}{3}xe^{3x}\Big|_0^1 - \frac{1}{9}e^{3x}\Big|_0^1$$

$$= \frac{1}{3}e^3 - \frac{1}{9}e^3 + \frac{1}{9} = \frac{2}{9}e^3 + \frac{1}{9}$$

Basic trig integrals: $\cos x$, $\sin x$ → $u = \cos x$ or $u = \sin x$

move powers around to make things fit

$$\int_0^{\pi/2} \sin^3 x dx =$$

A. 2/3 B. 4/3 C. 0 D. 1/4 E. 1/3

$$\int_0^{\pi/2} \underbrace{\sin^2 x}_{1-\cos^2 x} \cdot \underbrace{\sin x \, dx}$$

if $u = \cos x$

then $du = \underbrace{-\sin x \, dx}_{\text{we have this}}$

$$= \int_0^{\pi/2} (1-\cos^2 x) \sin x \, dx$$

$u = \cos x$

$du = -\sin x \, dx$

$$= \int_1^0 -(1-u^2)\, du = -u + \frac{1}{3}u^3 \Big|_1^0 = 0 - \left(-1 + \frac{1}{3}\right) = \frac{2}{3}$$

or: $$-u + \frac{1}{3}u^3 \Big|_{x=0}^{x=\pi/2} = -\cos x + \frac{1}{3}\cos^3 x \Big|_0^{\pi/2} = \dots =$$

$$\int_0^{\pi/4} \sec^4 x \tan x \, dx =$$

A. 1 B. 1/3 C. 4/3 D. 3/4 E. 2/9

if $u = \tan x$ then $du = \sec^2 x \, dx$

$u = \sec x$ then $du = \sec x \tan x \, dx$

$$\int_0^{\pi/4} \sec^4 x \tan x \, dx = \int_0^{\pi/4} \underset{\substack{\downarrow \\ \tan^2 x + 1 \\ \shortparallel \\ u^2+1}}{\sec^2 x} \cdot \underset{\substack{\downarrow \\ u}}{\tan x} \cdot \underbrace{\sec^2 x \, dx}_{du \text{ if } u = \tan x}$$

$1 = \tan \frac{\pi}{4}$

$$= \int_0^{\cancel{\pi/4}\ 1} (u^2+1)(u) \, du = \int_0^{\cancel{\pi/4}\ 1} (u^3+u) du = \frac{u^4}{4} + \frac{u^2}{2} \Big|_0^1$$

$$= \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$$

$$\int_0^{\pi/4} \sec^4 x \tan x \, dx$$

if $u = \sec x$

then $du = \sec x \tan x \, dx$

$$= \int_0^{\pi/4} \sec^3 x \; \sec x \tan x \, dx$$

$$= \int_1^{\sqrt{2}} u^3 \, du = \ldots$$

$$\int \frac{dx}{\sqrt{9-4x^2}} =$$

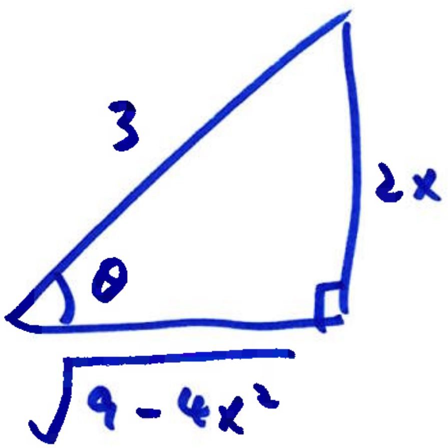
A. $\sec^{-1}\left(\frac{3x}{2}\right)+C$ B. $\frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right)+C$ C. $\tan^{-1}\left(\frac{2x}{3}\right)+C$ D. $\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)+C$

E. $\sqrt{9-4x^2}+\tan^{-1}\left(\frac{2x}{3}\right)+C$

trig subs. $\sqrt{9-4x^2}$

triangle w/ sides

$\sqrt{9-\underbrace{4x^2}_{(2x)^2}}$, 3, 2x



hypotenuse : 3

adjacent : $\sqrt{9-4x^2}$ (contains constant)

$\sin\theta = \frac{2x}{3}$ $x = \frac{3}{2}\sin\theta$

$dx = \frac{3}{2}\cos\theta\, d\theta$

$$\int \frac{dx}{\sqrt{9-4x^2}} = \int \frac{\frac{3}{2}\cos\theta\, d\theta}{\sqrt{9-4\left(\frac{9}{4}\sin^2\theta\right)}}\, dx$$

$$= \int \frac{\frac{3}{2}\cos\theta}{\sqrt{9\underbrace{(1-\sin^2\theta)}_{\cos^2\theta}}}\,d\theta = \int \frac{\frac{3}{2}\cancel{\cos}\theta}{3\cancel{\cos}\theta}\,d\theta$$

$$= \int \frac{1}{2}\,d\theta = \frac{1}{2}\theta + c = \frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right) + c$$

back to $\sin\theta = \frac{2}{3}x$

$$\theta = \sin^{-1}\left(\frac{2}{3}x\right)$$

$$\int \frac{x+1}{x^3-2x^2+x}\,dx =$$

A. $\ln|x| + \ln|x-1| + C$ B. $\ln|x| - \ln|x-1| + C$ C. $\ln|x| - \dfrac{2}{x-1} + C$

D. $\ln|x-1| - \dfrac{2}{x-1} + C$ E. $\ln|x| - \ln|x-1| - \dfrac{2}{x-1} + C$

partial fraction

$$\frac{x+1}{x(x^2-2x+1)} = \frac{x+1}{\underbrace{(x)}_{\text{linear}}\underbrace{(x-1)(x-1)}_{\text{repeated linear}}} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

mult. by $(x)(x-1)(x-1)$

$$x+1 = A(x-1)^2 + B(x)(x-1) + C(x)$$

$$= A(x^2-2x+1) + B(x^2-x) + Cx$$

$$0x^2 + 1x + 1 = (A+B)x^2 + (-2A-B+C)x + A$$

So,
$$\begin{cases} A+B=0 \\ -2A-B+C=1 \end{cases}$$

$A=1$

$B=-1$

$$-2(1)-(-1)+C=1$$

$$-2+1+C=1$$

$$C=2$$

$$\int\left(\frac{1}{x}-\frac{1}{x-1}+\frac{2}{(x-1)^2}\right)dx$$

$$=\int\frac{1}{x}\,dx-\int\frac{1}{x-1}\,dx+2\int\frac{1}{(x-1)^2}\,dx$$

$u=x-1$

$du=dx$

$$=\int\frac{1}{x}\,dx-\int\frac{1}{x-1}\,dx+2\int\frac{1}{u^2}\,du$$

$$=\ln|x|-\ln|x-1|-\frac{2}{u}+C=\ln|x|-\ln|x-1|-\frac{2}{x-1}+C$$

Indicate convergence or divergence for each of the following improper integrals:

$$\text{(I)} \int_2^\infty \frac{1}{(x-1)^2}\,dx \qquad \text{(II)} \int_0^2 \frac{1}{(x-1)^2}\,dx \qquad \text{(III)} \int_0^1 \frac{\ln x}{x}\,dx$$

conv. div. div.

A. I converges, II and III diverge. B. II converges, I and III diverge. C. I and III converge, II diverges. D. I and II converge, III diverges. E. I, II and III diverge.

I. $\lim_{a\to\infty} \int_2^a \frac{1}{(x-1)^2}\,dx = \lim_{a\to\infty} -\frac{1}{(x-1)}\Big|_2^a$

$= \lim_{a\to\infty} \underbrace{-\frac{1}{a-1}}_{\to 0 \text{ as } a\to\infty} + \frac{1}{1} = 1$

II. trouble at $x=1$

$\int_0^1 \frac{1}{(x-1)^2}\,dx + \int_1^2 \frac{1}{(x-1)^2}\,dx$

$= \lim_{a\to 1^-} \int_0^a \frac{1}{(x-1)^2}\,dx + \lim_{b\to 1^+} \int_b^2 \frac{1}{(x-1)^2}\,dx$

$$= \lim_{a\to 1^-} -\frac{1}{x-1}\Big|_0^a + \lim_{b\to 1^+} -\frac{1}{x-1}\Big|_b^2$$

$$= \lim_{a\to 1^-}\left(-\frac{1}{a-1}-1\right) + \underbrace{\lim_{b\to 1^+}\left(-1+\frac{1}{b-1}\right)}$$

as $a \to 1^-$

$$\frac{1}{a-1} = \frac{1}{\text{small negative}} = -\infty$$

$$= \infty + \text{whatever}$$

one part diverges, so whole thing diverges

IV. $\displaystyle\int_0^1 \frac{\ln x}{x}\,dx = \lim_{a\to 0^+}\int_a^1 \frac{\ln x}{x}\,dx$ $\quad u = \ln x$

$du = \frac{1}{x}\,dx$

$$= \lim_{a\to 0^+}\int_{\ln a}^0 u\,du = \lim_{a\to 0^+}\frac{1}{2}u^2\Big|_{\ln a}^0 = \lim_{a\to 0^+} 0 - \frac{1}{2}(\ln a)^2$$

$-\infty$