If $L = \sum_{n=1}^{\infty} \frac{1}{2^n} + \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n}$, then $L =$

A. 1/3 B. 2/3 C. 1 D. 4/3 E. 5/3

geometric series: $\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + ar^3 + \cdots = \frac{a}{1-r}$ ← first term of series

$|r|<1$ ← common ratio

$$\sum_{n=1}^{\infty} \frac{1}{2^n} = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots = \frac{1/2}{1-1/2} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n} = \sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n = 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \cdots$$

$$= \frac{1}{1-(-1/2)} = \frac{1}{3/2} = \frac{2}{3}$$

$$L = 1 + \frac{2}{3} = \frac{5}{3}$$

Find all values of p for which $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(n^2+1)^p}$ converges.

A. $p > 1$ B. $p \leq 1$ C. $p \geq 1$ D. $p > 1/2$ E. $p \leq 1/2$

p-series: $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^p}$ converges if $p > 1$

$\displaystyle\sum_{n=1}^{\infty} \frac{1}{(n^2+1)^p}$ when n is large becomes $\displaystyle\sum \frac{1}{(n^2)^p} = \sum \frac{1}{n^{2p}}$

converges if $2p > 1$

$p > 1/2$

Which of the following series converge conditionally?

$$\cancel{(\mathrm{I})}\ \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \qquad \cancel{(\mathrm{II})}\ \sum_{n=2}^{\infty} \frac{(-1)^n n}{\ln n} \qquad \cancel{(\mathrm{III})}\ \sum_{n=1}^{\infty} \frac{(-1)^n n}{e^n}$$

A. II only. B. I and III only. C. I and II only. D. All three. E. None of them.

conditionally : given $\sum a_k$, $\sum |a_k|$ diverges

but $\sum a_k$ converges

absolutely : given $\sum a_k$, both $\sum a_k$ and $\sum |a_k|$ converge

I. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ converges, because $\lim_{n\to\infty} \frac{(-1)^n}{n^2} = 0$ and $\frac{1}{n^2}$ decreases

(alternating series test)

$\sum_{n=1}^{\infty} \left| \frac{(-1)^n}{n^2} \right| = \sum_{n=1}^{\infty} \frac{1}{n^2}$ converges, $p > 1$

so I converges absolutely, not conditionally

II. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n n}{\ln n}$

divergence test : $\displaystyle\lim_{n\to\infty} \frac{n}{\ln n} = \lim_{n\to\infty} \frac{1}{\frac{1}{n}} = \lim_{n\to\infty} n \neq 0$

does not converge

III. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{e^n}$ by alt. series test, this converges

$$\sum_{n=1}^{\infty} \left| \frac{(-1)^n n}{e^n} \right| = \sum_{n=1}^{\infty} \frac{n}{e^n}$$

ratio test : $\displaystyle\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$ → if $=1$ inconclusive (same as in root test)

$$\lim_{n\to\infty} \left| \frac{\frac{n+1}{e^{n+1}}}{\frac{n}{e^n}} \right| = \lim_{n\to\infty} \frac{n+1}{e^{n+1}} \cdot \frac{e^n}{n} = \lim_{n\to\infty} \frac{n+1}{n} \cdot \frac{e^n}{e^{n+1}}$$

$$= \lim_{n\to\infty} \frac{n+1}{n} \cdot \frac{1}{e} = \frac{1}{e} < 1$$ conv. abs.

Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{n}{5^n}(x-2)^n$.

A. $-5 < x < 5$ B. $3 < x < 7$ C. $-2 < x < 2$ D. $-3 \leq x < 7$ E. $-3 < x < 7$

Ratio Test handles this well

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{\frac{n+1}{5^{n+1}}(x-2)^{n+1}}{\frac{n}{5^n}(x-2)^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{n+1}{5^{n+1}} \cdot \frac{5^n}{n} \cdot \frac{(x-2)^{n+1}}{(x-2)^n} \right| = \lim_{n\to\infty} \left| \overbrace{\frac{n+1}{n}}^{\to 1} \cdot \overbrace{\frac{5^n}{5^{n+1}}}^{\to \frac{1}{5}} \cdot \frac{(x-2)^{n+1}}{(x-2)^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(x-2)}{5} \right| < 1 \qquad \left| \frac{x-2}{5} \right| < 1$$

so $-1 < \frac{x-2}{5} < 1$

$-5 < x-2 < 5$ $\longrightarrow$ $-3 < x < 7$

not done!

check end values (ratio = 1 at ends)

$$\sum_{n=1}^{\infty} \frac{n}{5^n}(x-2)^n$$

at $x=-3$, series becomes $\sum_{n=1}^{\infty} \frac{n}{5^n}(-5)^n = \sum_{n=1}^{\infty} (-1)^n n$ diverges

at $x=7$, series becomes $\sum_{n=1}^{\infty} \frac{n}{5^n} 5^n = \sum_{n=1}^{\infty} n$ diverges

Use the power series representation of $\sin x$ to find the first three terms of the Maclaurin series of $f(x) = x\sin(x^2)$

A. $x^3 + \frac{x^7}{3!} + \frac{x^{11}}{5!}$ B. $x + \frac{x^3}{3} + \frac{x^5}{5}$ C. $x^3 - \frac{x^7}{3!} + \frac{x^{11}}{5!}$ D. $x - \frac{x^3}{3} + \frac{x^5}{5}$ E. $x^3 - \frac{x^7}{3} + \frac{x^{11}}{5}$

on exam, we will give $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$

$$= \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$$

$$\sin(x^2) = x^2 - \frac{(x^2)^3}{3!} + \frac{(x^2)^5}{5!} - \cdots = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \cdots$$

$$x\sin(x^2) = x^3 - \frac{x^7}{3!} + \frac{x^{11}}{5!} - \cdots$$

16. Recall that $\cos x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}$ for all x, and $\tan^{-1} x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1}$ for $|x| \leq 1$.

Find the limit

$$\lim_{x \to 0} \frac{\cos(x^3) + \tan^{-1}\left(\frac{x^6}{2}\right) - 1}{x^{12}}$$

A. $\frac{1}{4}$

B. $-\frac{7}{24}$

C. ∞

D. $\frac{1}{24}$

E. $-\frac{1}{3}$

F. 0

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$$

$$\cos(x^3) = 1 - \frac{x^6}{2!} + \frac{x^{12}}{4!} - \frac{x^{18}}{6!} + \cdots$$

$$\tan^{-1}\left(\frac{x^6}{2}\right) = \frac{x^6}{2} - \frac{x^{18}}{24} + \frac{x^{30}}{5 \cdot 32} - \cdots$$

$$\cos(x^3) + \tan^{-1}\left(\frac{x^6}{2}\right) - 1 = \left(\cancel{1} - \cancel{\frac{x^6}{2!}} + \frac{x^{12}}{4!} - \frac{x^{18}}{6!} + \cdots\right) + \left(\cancel{\frac{x^6}{2}} - \frac{x^{18}}{24} + \frac{x^{30}}{5 \cdot 32} - \cdots\right) - \cancel{1}$$

$$= \frac{x^{12}}{4!} - \frac{x^{18}}{6!} - \frac{x^{18}}{24} + \frac{x^{30}}{5 \cdot 32} + \text{bunch of } x^n \quad n \geq 30$$

$$\frac{\cos(x^3)+\tan^{-1}\left(\frac{x^6}{2}\right)-1}{x^{12}}$$

$$= \frac{\frac{x^{12}}{4!} - \frac{x^{18}}{6!} + \cdots}{x^{12}}$$

$x^n \quad n \geq 18$

$$= \underbrace{\frac{1}{4!} + \cdots}$$

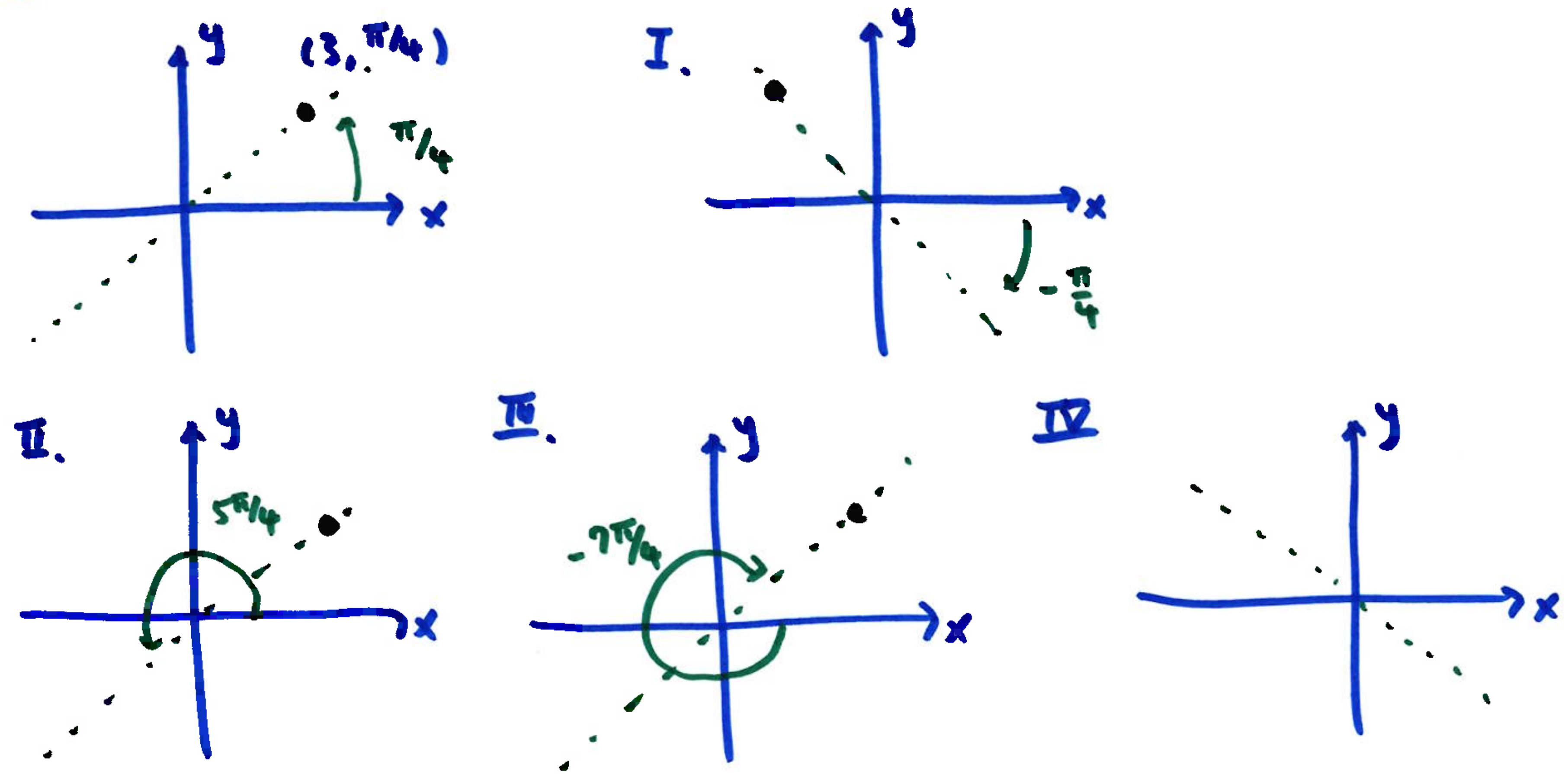
has x

$$\lim_{x \to 0}\left(\sim \right) = \frac{1}{4!} = \frac{1}{24}$$

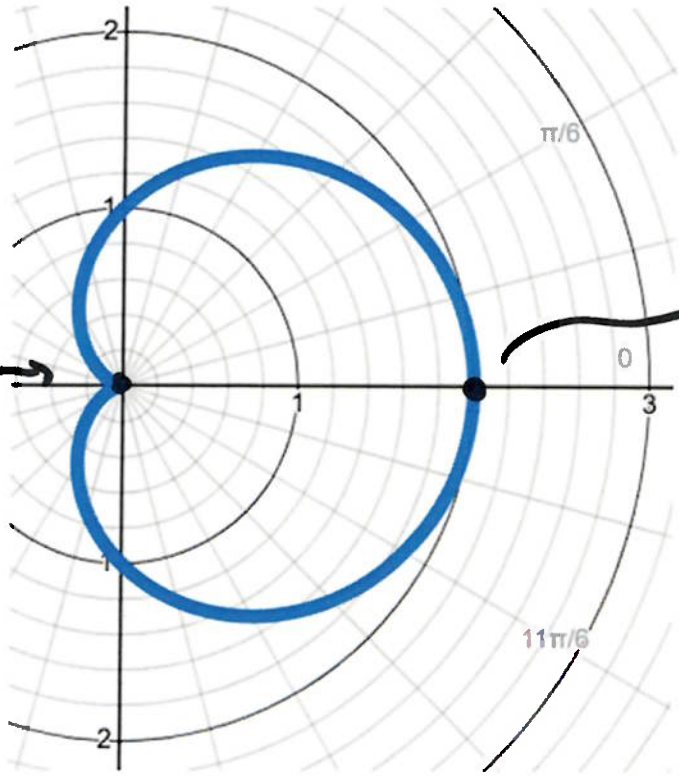
A point P has polar coordinates $(3, \pi/4)$. Which of the following are also polar coordinates of P?

(I) $(-3, -\pi/4)$ (II) $(-3, 5\pi/4)$ (III) $(3, -7\pi/4)$ (IV) $(3, -5\pi/4)$

A. I and II only. B. I and III only. C. I and IV only.
D. II and III only. E. II and IV only.



Which of the following polar equations describes the plot?



$r=2$ at $\theta=0$ or 2π

$\theta=\pi$
$r=0$

~~A.~~ $r = 2$ circle radius 2

B. $r = 1 + \cos(\theta)$ ⟶ at $\theta=0$, $r = 1+\cos 0 = 2$ $\theta = 2\pi$ $r=2$
$\theta=\pi$, $r = 1-1=0$

~~C.~~ $r = 2\cos(\theta)$ → $\theta=0$ $r=2$, ~~for~~ $\theta=2\pi$ $r=2$ ↳ $\theta=\pi$, $r=-2$

~~D.~~ $r = 1 - 2\cos(\theta)$ → $\theta=0$ $r=-1$

~~E.~~ $\theta = \frac{\pi}{4}$ line