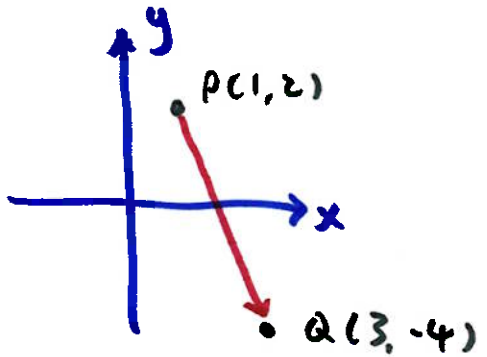
13.1 Vectors

scalars: numbers such as $5, \pi, e, 133$, etc
magnitude only, no direction

vectors: magnitude and direction
e.g. wind 25 mph from NE

vectors also tell us the relative position from one point to another

for example, P(1, 2), to Q(3, −4)



$\vec{PQ} = \langle 3-1, -4-2 \rangle = \langle 2, -6 \rangle$

end x minus start x; y

6 units down (negative)

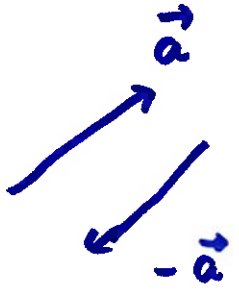
vector from P to Q

2 unit RIGHT (positive)

Q to P $\quad \vec{QP} = \langle -2, 6 \rangle$

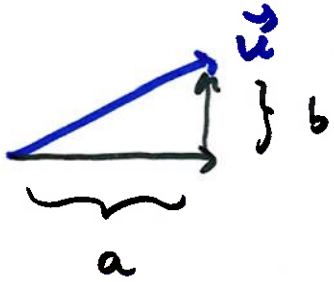
$\vec{QP} = -\vec{PQ}$

minus sign reverses direction



magnitude / length

- $\vec{u} = \langle a, b \rangle$



length : $|\vec{u}| = \sqrt{a^2 + b^2}$

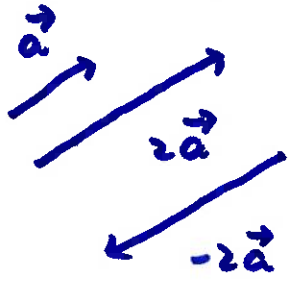
$\vec{PQ} = \langle -2, 6 \rangle \qquad |\vec{PQ}| = \sqrt{(-2)^2 + (6)^2} = \sqrt{40}$

addition / subtraction

$\vec{u} = \langle 1, 2 \rangle \qquad \vec{v} = \langle 0, 3 \rangle$

$\vec{u} + \vec{v} = \langle 1+0, 2+3 \rangle = \langle 1, 5 \rangle$

$\vec{u} - \vec{v} = \langle 1-0, 2-3 \rangle = \langle 1, -1 \rangle$



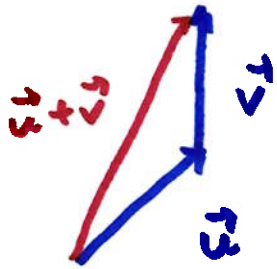
$2\vec{u} - 3\vec{v} = 2\langle 1, 2 \rangle - 3\langle 0, 3 \rangle$

$= \langle 2, 4 \rangle - \langle 0, 9 \rangle$

$= \langle 2, -5 \rangle$

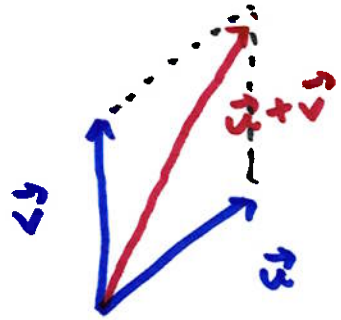
graphically:

$\vec{u}+\vec{v}$:



"triangle rule"

$\vec{u}+\vec{v}$:



"parallelogram rule"

unit vector : vector of length or magnitude of 1

$\vec{u} = \langle 3, 2 \rangle \qquad |\vec{u}| = \sqrt{3^2 + 2^2} = \sqrt{13} \neq 1$

So, $\vec{u}$ is NOT a unit vector

but $\vec{v} = \dfrac{\vec{u}}{|\vec{u}|} = \dfrac{\langle 3, 2 \rangle}{\sqrt{13}} = \left\langle \dfrac{3}{\sqrt{13}}, \dfrac{2}{\sqrt{13}} \right\rangle$

$$|\vec{v}| = \sqrt{\left(\frac{3}{\sqrt{13}}\right)^2 + \left(\frac{2}{\sqrt{13}}\right)^2} = \sqrt{\frac{9}{13} + \frac{4}{13}} = \sqrt{\frac{13}{13}} = 1$$

So, in general, $\dfrac{\vec{a}}{|\vec{a}|}$ is a unit vector $\hat{}$

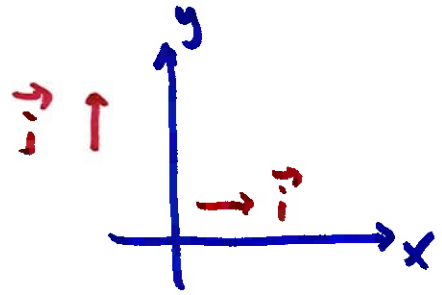
unit vector in opposite direction of $\vec{b}$?

$$-\frac{\vec{b}}{|\vec{b}|}$$

vector length of 3, in opposite direction of $\vec{b}$? $-3\dfrac{\vec{b}}{|\vec{b}|}$

Special unit vectors: $\vec{i} = \langle 1, 0 \rangle$ unit vector in x direction

$\vec{j} = \langle 0, 1 \rangle$ " " " y "



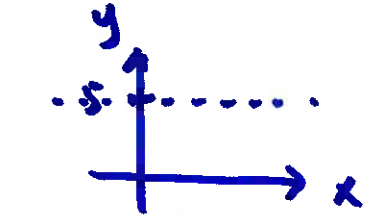
13.2 Vectors in 3D

P(1, 2, 3) Q(4, 6, 8)

$\vec{PQ} = \langle 4-1, 6-2, 8-3 \rangle = \langle 3, 4, 5 \rangle = 3\vec{i} + 4\vec{j} + 5\vec{k}$

$\langle 0, 0, 1 \rangle$

shapes: line in 2D: $y = 5$

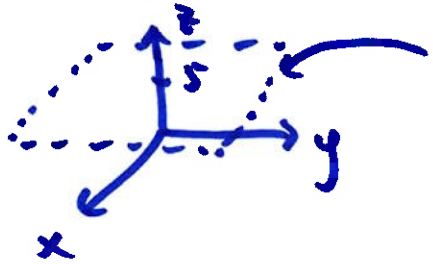


all points w/ $y = 5$

in 3D, it's a plane

$z = 5$

plane



all points w/ $z = 5$

$(x, y, 5)$