8.2 Integration By Parts

NOT on exam 1

HW includes 8.1 (Basic integration)

Integration by substitution: Chain rule in reverse

Integration by parts: Product rule in reverse

u, v both functions of x

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

in the differential form

$$d(uv) = u\,dv + v\,du$$

$$u\,dv = d(uv) - v\,du$$

integrate

$$\int u\,dv = \int d(uv) - \int v\,du$$

$$\int u\,dv = uv - \int v\,du$$

Integration by parts

$$\boxed{\int u\,dv = uv - \int v\,du}$$

given integral, identify u and dv, then use $uv - \int v\,du$

example

$$\int x \cdot \ln x \, dx$$

not something we can integrate directly

substitution doesn't work either

now try by parts

identify u, dv

$$\int x \cdot \ln x \, dx$$

two parts to consider: x and $\ln x$

u: has a simpler derivative

dv: easy to integrate

a good rule of thumb for order of choosing u

LIATE

logarithmic

inverse trig

algebraic (e.g. x^2)

trig

exponential

pick in that order

back to $\int \underline{x} \cdot \underline{\ln x}\, dx$

algebraic

logarithmic

LIATE: pick $u = \ln x$ since it's L
which is before A (x)

$\boxed{u = \ln x}$ $\quad dv = x\,dx$ → any leftover after choosing u

differentiate

$\frac{du}{dx} = \frac{1}{x}$

$\boxed{du = \frac{1}{x}\,dx}$

integrate

$V = \int dv$

$\boxed{v = \int x\,dx = \frac{1}{2}x^2}$ $\quad$ don't need $+C$

formula: $\int u\,dv = uv - \int v\,du$

$$= \overset{u}{(\ln x)}\overset{v}{\left(\tfrac{1}{2}x^2\right)} - \int \overset{v}{\left(\tfrac{1}{2}x^2\right)}\overset{du}{\left(\tfrac{1}{x}\,dx\right)}$$

$$= \frac{1}{2}x^2 \ln x - \int \frac{1}{2}x\,dx = \boxed{\frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C}$$

to check: take derivative
it must match $x \ln x$

example $\int x \cos x \, dx$

parts: $\underline{\underline{x}}$, $\underline{\underline{\cos x}}$

algebraic (A)

trig (T)

order for u: LIATE

A before T so we choose $u = x$

deriv. $u = x$, $du = dx$

integrate $dv = \cos x \, dx$, $v = \sin x$

+c is not needed here

$$uv - \int v \, du$$

$$= x \sin x - \int \sin x \, dx = x \sin x - (-\cos x) + C$$

$$= \boxed{x \sin x + \cos x + c}$$

definite integral $\int_0^{\pi/2} x \cos x \, dx$

pick u, dv as usual

then $uv \Big|_0^{\pi/2} - \int_0^{\pi/2} v \, du = x \sin x \Big|_0^{\pi/2} - \int_0^{\pi/2} \sin x \, dx$

$$= \frac{\pi}{2}\underbrace{\sin\frac{\pi}{2}}_{1} + \cos x\Big|_0^{\pi/2} = \frac{\pi}{2} + \underbrace{\cos\left(\frac{\pi}{2}\right)}_{0} - \underbrace{\cos(0)}_{1} = \boxed{\frac{\pi}{2} - 1}$$

<u>example</u> $\int \underset{A}{x^2}\, \underset{T}{\cos x}\, dx$

LIATE

so $u = x^2$ (A before T)

$u = x^2 \qquad dv = \cos x\, dv$

$du = 2x\,dx \qquad v = \sin x$

$uv - \int v\, du$

$$= x^2 \sin x - \int (\sin x)(2x)\,dx$$

$$= x^2 \sin x - 2\underbrace{\int x \cdot \sin x\, dx}$$

by parts again, ~~xx~~ $x \to A$ $\quad \sin x \to T$

$U = x \qquad dV = \sin x\, dx$

$dU = dx \qquad V = -\cos x$

$UV - \int V\, dU$

$= x^2 \sin x - 2\left(uv - \int v\,du\right)$

$= x^2 \sin x - 2\left(-x\cos x - \int -\cos x\,dx\right)$

$= x^2 \sin x - 2\left(-x\cos x + \int \cos x\,dx\right)$

$= x^2 \sin x - 2\left(-x\cos x + \sin x\right) + C$

$= \boxed{x^2 \sin x + 2x\cos x - 2\sin x + C}$

$\int x^n \cos x\,dx \rightarrow n$ rounds

<u>example</u> $\displaystyle\int \underset{E}{\underline{\underline{e^x}}} \, \underset{T}{\underline{\underline{\cos x}}} \, dx$

LIATE

T before E so, $u = \cos x$

$u = \cos x \qquad dv = e^x dx$

$du = -\sin x \, dx \qquad v = e^x$

$$\int e^x \cos x \, dx = uv - \int v \, du$$

$$= e^x \cos x - \int e^x \cdot -\sin x \, dx$$

$$= e^x \cos x + \underbrace{\int e^x \sin x \, dx}_{\text{by parts}}$$

$U = \sin x \qquad dV = e^x dx$

$dU = \cos x \, dx \qquad V = e^x$

$$\underbrace{\int e^x \cos x \, dx}_{?} = e^x \cos x + \left(UV - \int V \, dU \right)$$

$$= e^x \cos x + e^x \sin x - \underbrace{\int e^x \cos x \, dx}_{\text{orig. integral}}$$

by parts in infinite loop

STOP !

$$\boxed{\int e^x \cos x\,dx} = e^x \cos x + e^x \sin x - \boxed{\int e^x \cos x\,dx}$$

add $\int e^x \cos x\,dx$ to both sides

$$2\boxed{\int e^x \cos x\,dx} = e^x(\cos x + \sin x)$$

so, $\int e^x \cos x\,dx = \boxed{\frac{1}{2} e^x(\cos x + \sin x) + C}$

this is an example of a case where going against LIATE still works:

$u = e^x \qquad dv = \cos x\,dx$

$du = e^x dx \qquad v = \sin x$

EXAM 1 REVIEW SESSION

SEPT 18th
(Monday)
5:30pm-7:30pm
MATH 175