

13.2 Vectors in 3D (part 2)

vectors can be used to build shapes
for example, sphere



vector from center (h, k, l)

to point on sphere (x, y, z)

must have a magnitude of r (radius)

$c(h, k, l)$

vector from center to point: $\langle x-h, y-k, z-l \rangle$

$$|\langle x-h, y-k, z-l \rangle| = r$$

$$\sqrt{(x-h)^2 + (y-k)^2 + (z-l)^2} = r$$

$$(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2$$

standard form of eq. of sphere

example

Find eq. of sphere whose diameter has endpoints

$$P(-2, 3, 6), Q(4, -7, 5)$$



need center, radius

center: midpoint between, P, Q

average of, x, y, z coordinates

$$C\left(\frac{-2+4}{2}, \frac{3-7}{2}, \frac{6+5}{2}\right)$$

$$= C\left(1, -2, \frac{11}{2}\right)$$

radius: half distance of $\vec{PQ}$

$$\vec{PQ} = \langle 6, -10, -1 \rangle$$

$$|\vec{PQ}| = \sqrt{36+100+1} = \sqrt{137} \quad \text{radius} = \frac{\sqrt{137}}{2}$$

eg:

$$\left[(x-1)^2 + (y+2)^2 + \left(z - \frac{11}{2}\right)^2 = \left(\frac{\sqrt{137}}{2}\right)^2 \right. \\ \left. = \frac{137}{4} \right]$$

example

Find center and radius of

$$x^2 + y^2 + z^2 - 14x + 16y - 10z + 102 = 0$$

not in standard form, let's turn it into the standard form

complete the square:

$$x^2 - 14x + \frac{49}{\underbrace{\quad}} + y^2 + 16y + \frac{64}{\underbrace{\quad}} + z^2 - 10z + \frac{25}{\underbrace{\quad}} = -102$$

half of this squared half of this squared

+49 +64 +25

$$x^2 - 14x + 49 + y^2 + 16y + 64 + z^2 - 10z + 25 = 36$$

$$(x-7)^2 + (y+8)^2 + (z-5)^2 = 6^2$$

center: (7, -8, 5)

radius: 6

complete the square: coefficient of squared terms must be 1

$$(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2$$

all vectors from center to surface have length r

what about $(x-h)^2 + (y-k)^2 + (z-l)^2 \leq r^2$

all vectors from center have length $\leq r$

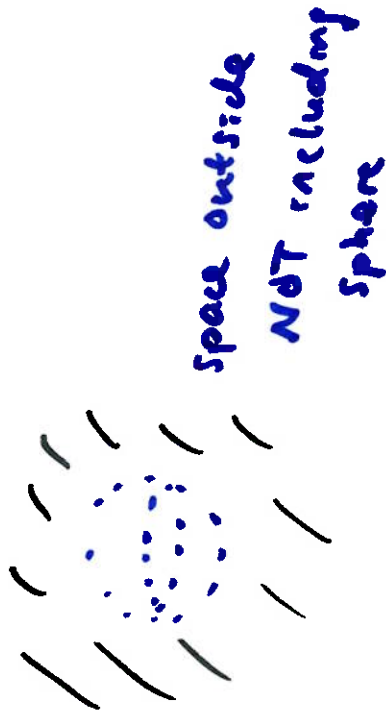
sphere + all space inside $\rightarrow$ solid ball

what about $(x-h)^2 + (y-k)^2 + (z-l)^2 < r^2$

ball w/o the spherical "skin"

$$(x-h)^2 + (y-k)^2 + (z-l)^2 > r^2$$

vectors have lengths $> r$



$$9 \leq (x-1)^2 + (y+2)^2 + (z-5)^2 \leq 25$$

↓ center: (1, -2, 5) ↓

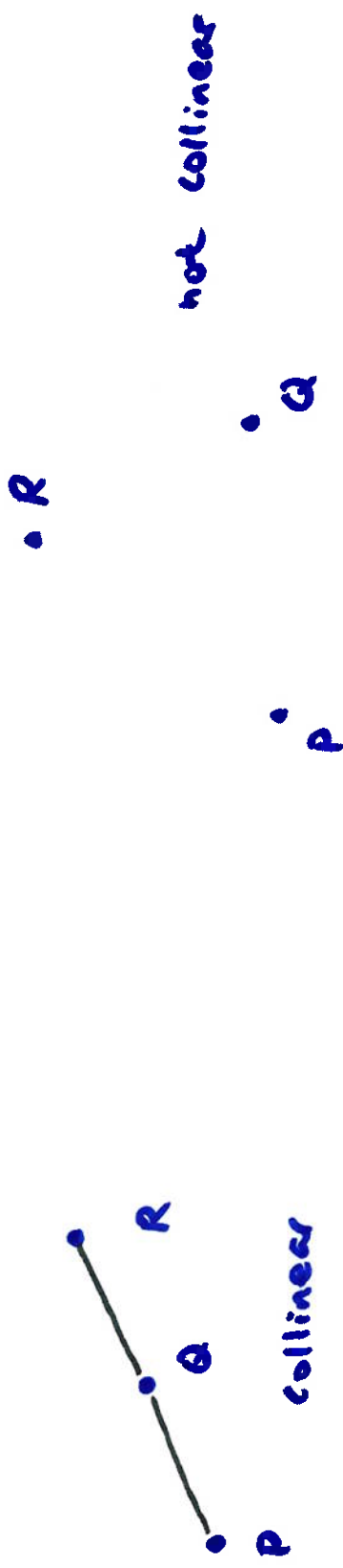
radius 3

radius 5

vectors from center have magnitudes between 3 and 5



Given 3 points in 3D space ($\mathbb{R}^3$), how do we know if they lie along a line? (collinear)



note: distances between them are not relevant for collinear



example $P(1, 3, 2)$, $Q(5, 1, 3)$, $R(x, 2, z)$ are collinear

find x, z

$$\vec{PQ} = \langle 4, -2, 1 \rangle$$

$$\vec{PR} = \langle x-1, -1, z-2 \rangle$$

$$\vec{PQ} \parallel \vec{PR} \rightarrow \vec{PQ} = k \vec{PR}$$

$$\langle 4, -2, 1 \rangle = k \langle x-1, -1, z-2 \rangle \text{ vector equation}$$

turns into scalar equations by equating components

$$x: 4 = k(x-1)$$

$$y: -2 = k(-1) \rightarrow k = 2 \text{ now use this in the other equations to find } x, z$$

$$z: 1 = k(z-2)$$

$$x: 4 = 2(x-1) \rightarrow \boxed{x = 3}$$

$$z: 1 = 2(z-2) \rightarrow \boxed{z = \frac{5}{2}}$$