6.7 Physical Applications (part 1)

long thin bar/wire

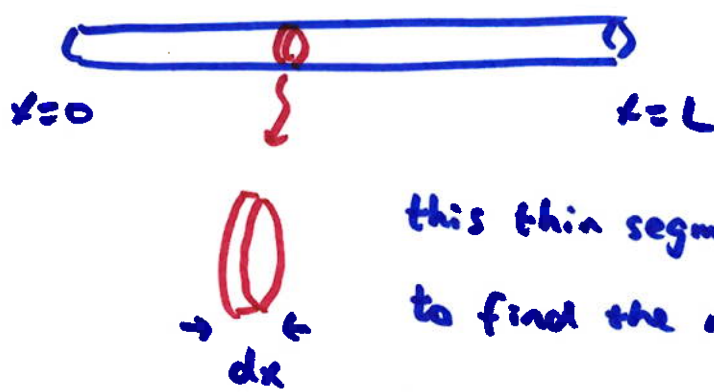
length is L m

it's very thin so only the length matters

if the density is constant : ρ (rho)

then mass is $m = \rho \cdot L$

if density is not constant, say $\rho = \rho(x)$



this thin segment has mass $= \rho(x)\,dx$

to find the mass of entire thing

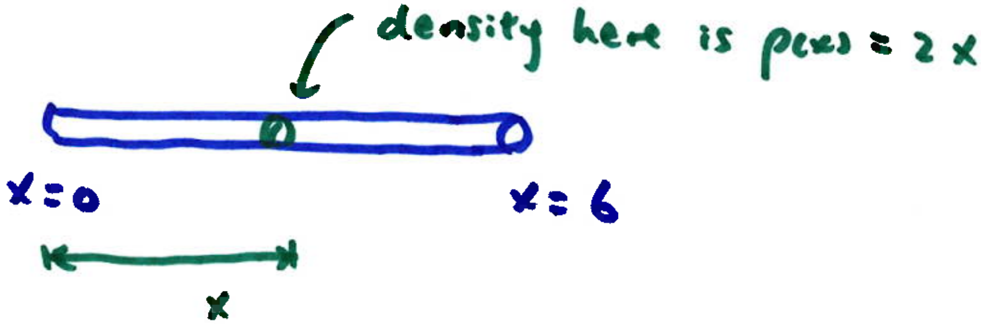
we accumulate these by integration

$$m = \int_0^L \rho(x)\,dx$$

<u>example</u> wire length 6 m

density is twice the distance from the left end

mass = ?



So mass of this wire is $\int_0^6 2x\,dx = 36$

follow-up question: what is the mass of the right half of the wire?

the accumulation starts at $x=3$ ends at $x=6$

$$\int_3^6 2x\,dx = 27$$

work = force · distance   this assumes force is constant

in many cases the force is not constant

for example, a spring

force on a spring : Hooke's Law

$$F = kx$$

- F: force
- k: spring constant (stiffness)
- x: deviation from the equilibrium position

the equilibrium position → no force is involved

the length of spring at equilibrium is natural length

the work in this case (non-constant force)

is
$$W = \int_a^b \text{force} \cdot dx = \int_a^b kx \, dx$$

a: starting length measured with respect to natural length

b: ending " " " " " " "

example Spring has natural length of 1 m

A force of 40 N stretches and holds it at 0.1 m from its equilibrium

Find: work done in compressing from natural length to a length of 0.5 m

~~best~~
~~additional~~
work done in compressing it by another 0.5 m

→ tells us the spring constant

$$F = kx$$

$$40 = k \cdot (0.1)$$ ← force of 40 N stretches it 0.1 m beyond equilibrium

$$k = 400$$

work to compress from natural $(x=0)$ to a length of 0.5 m $(x = -0.5)$

$$W = \int_0^{-0.5} 400x \, dx = \cdots = 50 \text{ N}\cdot\text{m (Joules)}$$

work to compress another 0.5 m

$$W = \int_{-0.5}^{-1} 400x \, dx = \dots = 150 \text{ J}$$

work done against gravity

if the change in height is small compared to the size Earth then we can consider the acceleration due to gravity as constant: $g = 9.8 \ m/s^2$

work = force · distance

work in moving a mass of 45 kg from the ground to a height of 60 m is $W = (45 \cdot g) \cdot 60 = 2700 g$

($g \to 9.8$)

just like with force, if distance is not constant, then an integration is involved.

example: work in winding up a chain or cable (distributed mass)