Let $f(x, y) = (x^2 + y^2)e^x$. The function has

A. a local max. and a local min. point

B. two local max. points

C. a local max. and a saddle point

D. two local max. points

E. a local min. and a saddle point

critical pts: $f_x = 0$, $f_y = 0$

$$f_x = (x^2+y^2)e^x + 2xe^x = 0$$

$$f_y = 2ye^x = 0 \rightarrow y = 0 \quad (e^x \neq 0)$$

sub $y=0$ into $f_x = 0$ equation

$$x^2e^x + 2xe^x = 0$$

$$xe^x(x+2) = 0 \rightarrow x = 0, \ x = -2$$

critical pts: $(0,0)$, $(-2,0)$ $\qquad D = f_{xx}f_{yy} - (f_{xy})^2$

$$f_{xx} = (x^2+y^2)e^x + 2xe^x + 2xe^x + 2e^x \qquad f_{yy} = 2e^x \qquad f_{xy} = f_{yx} = 2ye^x$$

$$D(0,0) = (2)(2) - 0 > 0, \quad f_{xx} > 2 \rightarrow (0,0) \text{ local min}$$

$$D(-2,0) = (4e^{-2} - 4e^{-2} - 4e^{-2} + 2e^{-2})(2e^{-2}) - 0$$

$$= -4e^{-4} < 0 \rightarrow \text{saddle pt at } (-2,0)$$

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3) The maximum of $f(x,y,z) = x + y + z$ subject to the constraint $(x-1)^2 + y^2 + z^2 = 1$ is

A) $1 + \sqrt{3}$

B) $1 - \sqrt{3}$

C) $\sqrt{3}$

D) $1 + 2\sqrt{3}$

E) $1 + 3\sqrt{3}$

$f(x,y,z) = x+y+z$

$g(x,y,z) = (x-1)^2+y^2+z^2-1=0$

$\vec{\nabla} f = \lambda \vec{\nabla} g$

solve for x, y, z using $g(x,y,z)$

$\vec{\nabla} f = \langle 1,1,1\rangle \quad \vec{\nabla} g = \langle 2(x-1), 2y, 2z\rangle$

$\langle 1,1,1\rangle = \lambda \langle 2(x-1), 2y, 2z\rangle$

$$\left.\begin{array}{l} 1 = \lambda \cdot 2(x-1) \\ 1 = \lambda\cdot 2y \\ 1 = \lambda \cdot 2z \end{array}\right\} \quad \left.\begin{array}{l} x-1=y \\ y = z \end{array}\right\}$$ sub out x and z in $g(x,y,z)$

$y^2+y^2+y^2=1 \qquad 3y^2=1 \qquad y^2 = \frac{1}{3}$

$y = \pm\frac{1}{\sqrt{3}}, \quad z = y = \pm \frac{1}{\sqrt{3}}, \quad x = 1+y = 1\pm\frac{1}{\sqrt{3}}$

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note $y = x - 1$, $y = z$ require x, y, z to have the same sign

so, the points of interest are: $\left(1+\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$, $\left(1-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$

$$f(x,y,z) = x+y+z$$

$$f\left(1+\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) = 1+\frac{3}{\sqrt{3}} = 1+\sqrt{3} \quad \text{max}$$

$$f\left(1-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right) = 1-\frac{3}{\sqrt{3}} = 1-\sqrt{3} \quad \text{min}$$

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Let D be the triangle with vertices $(0,4)$, $(1,0)$, and $(0,-2)$. Then $\int\int_D f(x,y)\,dA$ is

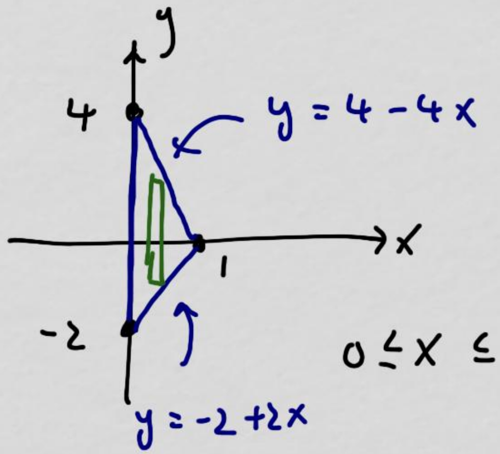
A. $\int_0^2 \int_{2-2x}^{4-4x} f(x,y)\,dy\,dx$

B. $\int_0^1 \int_{2x-2}^{4-4x} f(x,y)\,dy\,dx$

C. $\int_0^2 \int_{2+2x}^{4+4x} f(x,y)\,dy\,dx$

D. $\int_0^4 \int_{2-2x}^{4-4x} f(x,y)\,dy\,dx$

E. $\int_0^1 \int_{2-2x}^{4+4x} f(x,y)\,dy\,dx$



$0 \le x \le 1$

$2x-2 \le y \le 4-4x$

note all choices have the order dydx → Type I region

$$\iint_D f(x,y)\,dA = \int_0^1 \int_{2x-2}^{4-4x} f(x,y)\,dy\,dx$$

Question Help

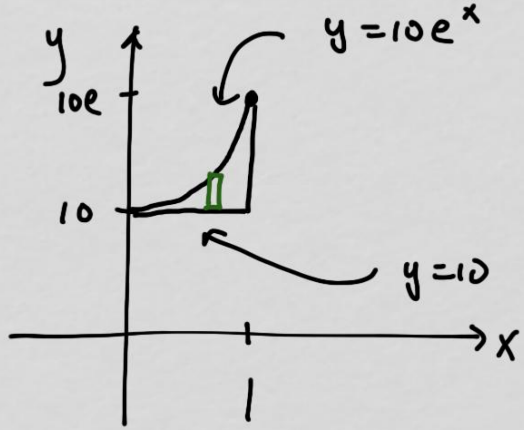
Reverse the order of integration in the following integral.

$$\int_0^1 \int_{10}^{10e^x} f(x,y)\,dy\,dx$$

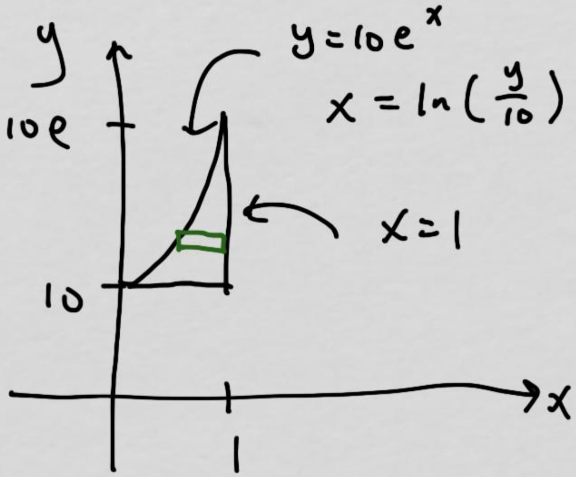
$dy\,dx \to$ Type I

$0 \le x \le 1$

$10 \le y \le 10e^x$



now switch to Type $\underline{II}$



$10 \le y \le 10e$

$\ln\left(\frac{y}{10}\right) \le x \le 1$

$$\boxed{\int_{10}^{10e} \int_{\ln(y/10)}^{1} f(x,y)\,dx\,dy}$$

Evaluate $\iiint_E z\, dV$ where E is bounded by $y^2 + z^2 = 4$ and the planes $x = 0$, $y = x$, and $z = 0$ in the first octant.

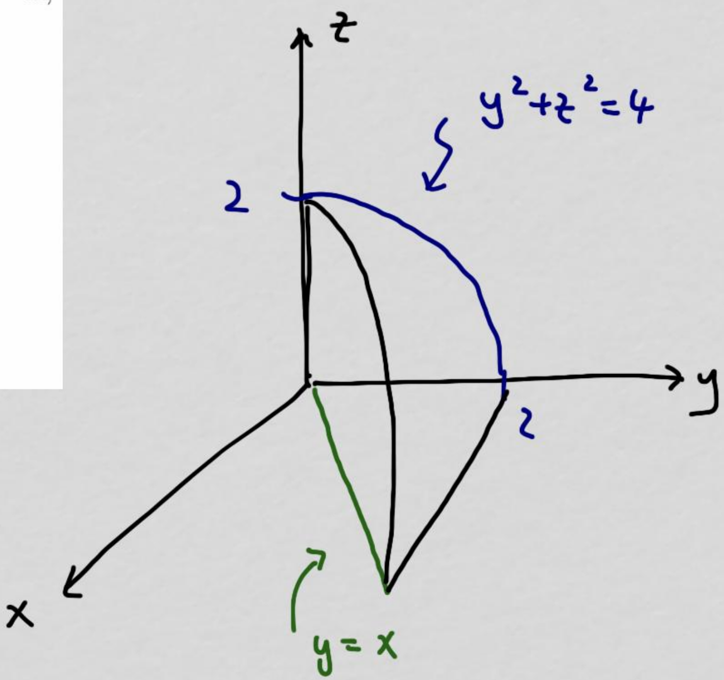
A. 1

B. 2

C. 4

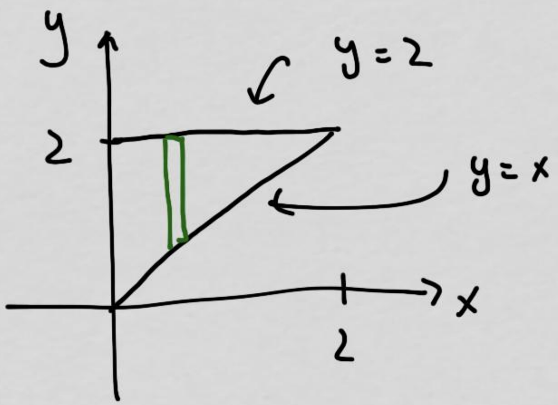
D. 8

E. 16



portion of cylinder $y^2+z^2=4$ chopped off by planes $y=x$, $z=0$, $x=0$

let the "floor" be xy-plane



$0 \le x \le 2$

$x \le y \le 2$

$0 \le z \le \sqrt{4-y^2}$ "ceiling"

↳ rearrangement of $y^2+z^2=4$

$$\int_0^2 \int_x^2 \int_0^{\sqrt{4-y^2}} z \, dz \, dy \, dx = \int_0^2 \int_x^2 \frac{1}{2} z^2 \Big|_{z=0}^{z=\sqrt{4-y^2}} dy \, dx$$

$$= \frac{1}{2} \int_0^2 \int_x^2 (4-y^2) \, dy \, dx = \frac{1}{2} \int_0^2 4y - \frac{1}{3} y^3 \Big|_{y=x}^{y=2} dx$$

$$= \frac{1}{2} \int_0^2 \left[4 \cdot 2 - \frac{1}{3}(2)^3\right] - \left[4 \cdot x - \frac{1}{3}(x)^3\right] dx$$

$$= \frac{1}{2} \int_0^2 \left(\frac{16}{3} - 4x + \frac{1}{3} x^3\right) dx = \frac{1}{2} \left(\frac{16}{3} x - 2x^2 + \frac{1}{12} x^4\right) \Big|_0^2$$

$$= \frac{1}{2} \left(\frac{32}{3} - 8 + \frac{16}{12}\right) = \frac{1}{2} \left(\frac{32}{3} + \frac{4}{3} - 8\right) = \frac{1}{2} (12 - 8) = \boxed{2}$$

Find the volume of the solid that is enclosed by $x^2+y^2+z^2=1$, $x^2+y^2+z^2=4$, and $z=\sqrt{x^2+y^2}$.

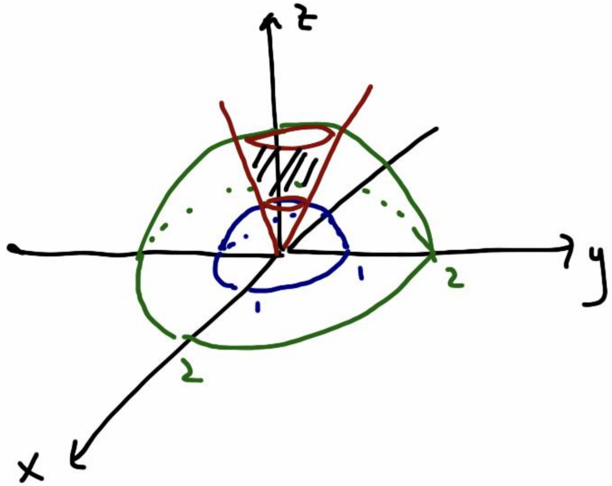
A. $\frac{14\pi}{3}\left(1-\frac{\sqrt{2}}{2}\right)$

B. $\frac{28\pi}{3}$

C. $\frac{14\pi}{3}\left(1+\frac{\sqrt{2}}{2}\right)$

D. $3\pi\left(1-\frac{\sqrt{2}}{2}\right)$

E. 3π



part of cone between 2 spheres (like a cork shape)

choice of coordinates: Cartesian, Cylindrical, Spherical

↳ best because we are dealing w/ spheres

$1 \le \rho \le 2$ (between spheres of radii 1 and 2)

$0 \le \theta \le 2\pi$ (all the way around about z-axis)

$0 \le \phi \le \frac{\pi}{4}$ (cone has slope of 1 on yz-plane, slope 1 = 45°)

$$\underset{\theta}{\int_0^{2\pi}} \underset{\phi}{\int_0^{\pi/4}} \underset{\rho}{\int_1^2} \underbrace{\rho^2 \sin\phi \, d\rho \, d\phi \, d\theta}_{dV} = \int_0^{2\pi}\int_0^{\pi/4} \frac{1}{3}\rho^3 \Big|_{\rho=1}^{\rho=2} \sin\phi \, d\phi \, d\theta$$

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$$= \int_0^{2\pi} \int_0^{\pi/4} \frac{1}{3}(7) \sin\phi \, d\phi \, d\theta = \frac{7}{3} \int_0^{2\pi} -\cos\phi \Big|_{\phi=0}^{\phi=\pi/4} d\theta$$

$$= \frac{7}{3} \int_0^{2\pi} \left(1 - \frac{1}{\sqrt{2}}\right) d\theta = \frac{7}{3}\left(1 - \frac{1}{\sqrt{2}}\right) \cdot 2\pi = \boxed{\frac{14\pi}{3}\left(1 - \frac{\sqrt{2}}{2}\right)}$$

Do NOT evaluate. Rewrite the integral in cylindrical coordinates.

$$\int_0^2 \int_0^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} \sqrt{x^2+y^2}\, dz\, dx\, dy$$

A. $\int_0^{\pi/2} \int_0^2 \int_r^{\sqrt{8-r^2}} r^2\, dz\, dr\, d\theta$

B. $\int_0^{\pi/2} \int_0^2 \int_r^{\sqrt{8-r^2}} r\, dz\, dr\, d\theta$

C. $\int_0^{\pi} \int_0^2 \int_r^{\sqrt{8-r^2}} r^2\, dz\, dr\, d\theta$

D. $\int_0^{\pi} \int_0^2 \int_r^{\sqrt{8-r^2}} r\, dz\, dr\, d\theta$

E. None of the above.

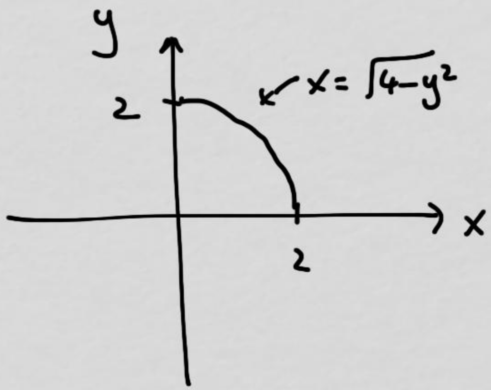
$\underbrace{dz\, dx\, dy}$

"floor" is xy-plane

(Type II)

$0 \le y \le 2$

$0 \le x \le \sqrt{4-y^2}$



$0 \le \theta \le \pi/2$ (QI)

$0 \le r \le 2$ (circle of radius 2)

"floor" is $z = \sqrt{x^2+y^2}$ (cone) and expressed w/ r, θ

$z = \sqrt{r^2} = r$

"ceiling" is $z = \sqrt{8-x^2-y^2}$ (sphere)

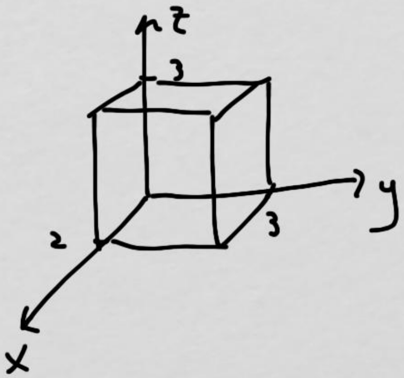
$z = \sqrt{8-r^2}$

$$\int_0^2 \int_0^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} \underbrace{\sqrt{x^2+y^2}}_{r} \; \underbrace{dz\,dx\,dy}_{dV}$$

$$= \underbrace{\int_0^{\pi/2}}_{\theta} \underbrace{\int_0^2}_{r} \underbrace{\int_r^{\sqrt{8-r^2}}}_{z} r \cdot \overbrace{r\,dz\,dr\,d\theta}^{dV}$$

Find the coordinates of the center of mass of the following solid with variable density.

$R = \{(x,y,z): 0 \le x \le 2,\ 0 \le y \le 3,\ 0 \le z \le 3\};\ \rho(x,y,z) = 3 + \frac{x}{2}$



$$\rho(x,y,z) = 3 + \frac{x}{2}$$

$$\bar{x} = \frac{1}{m} \iiint_D x\rho \, dV \qquad m = \iiint_D \rho \, dV$$

$$\bar{y} = \frac{1}{m} \iiint_D y\rho \, dV$$

$$\bar{z} = \frac{1}{m} \iiint_D z\rho \, dV$$

let's do $\bar{x}$ only.

$$m = \int_0^2 \int_0^3 \int_0^3 \left(3 + \tfrac{1}{2}x\right) dz \, dy \, dx$$

$$= \int_0^2 \int_0^3 3z + \tfrac{1}{2}xz \Big|_{z=0}^{z=3} dy \, dx$$

$$= \int_0^2 \int_0^3 9 + \tfrac{3}{2}x \; dy \, dx = \int_0^2 9y + \tfrac{3}{2}xy \Big|_{y=0}^{y=3} dx$$

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$$= \int_0^2 27 + \frac{9}{2}x \, dx = 27x + \frac{9}{4}x^2 \Big|_0^2 = 54 + 9 = 63$$

$$\bar{x} = \frac{1}{m} \iiint_D x \rho \, dV = \frac{1}{63} \int_0^2 \int_0^3 \int_0^3 x\left(3 + \frac{1}{2}x\right) dz\, dy\, dx$$

$$= \cdots = \frac{22}{21}$$

2D formulas: $m = \iint_R \rho \, dA$

$$\underline{\underline{\bar{x}}} = \frac{1}{m} \iint_R \underline{\underline{x}} \rho \, dA$$

$$\underline{\underline{\bar{y}}} = \frac{1}{m} \iint_R \underline{\underline{y}} \rho \, dA$$

Find the work done by the force field $\vec{F}(x,y) = 2x\sin(y)\vec{i} + 2y\vec{j}$ on a particle moving along the parabola $y = x^2$ from the point $(1,1)$ to the point $(2,4)$.

A. $17 - \cos(1) + \cos(4)$

B. $17 + \cos(1) - \cos(4)$

C. $15 + \sin(1) - \sin(4)$

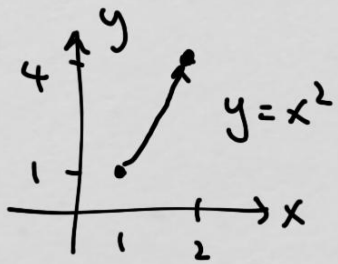
D. $15 + \cos(1) - \cos(4)$

E. $15 + \sin(1) - \cos(4)$

line integral $\int_C \vec{F}\cdot\vec{T}\,ds = \int_C \vec{F}\cdot\vec{r}\,'\,dt$

is also the work done by the force field $\vec{F}$ in moving something along C

C :



let $x = t$, then $y = x^2 = t^2$ $\quad 1 \le t \le 2$

$\vec{r}(t) = \langle t, t^2 \rangle \quad 1 \le t \le 2$

$\vec{F} = \langle 2x\sin y, 2y \rangle = \langle 2t\sin t^2, 2t^2 \rangle$

$\vec{r}\,' = \langle 1, 2t \rangle$

$W = \int_C \vec{F}\cdot\vec{r}\,'\,dt = \int_1^2 \langle 2t\sin t^2, 2t^2\rangle\cdot\langle 1, 2t\rangle\,dt = \int_1^2 (\underbrace{2t\sin t^2}_{u=t^2} + 4t^3)\,dt$

$= \dots = -\cos t^2\Big|_1^2 + t^4\Big|_1^2 = \boxed{\cos(1) - \cos(4) + 15}$

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13. If $\vec{F} = (3+2xy)\vec{i} + (x^2 - 3y^2)\vec{j}$ and $\vec{F} = \vec{\nabla} f$, find $\int_C \vec{\nabla} f \cdot d\vec{r}$ if the curve C is parametrized as $\vec{r}(t) = e^t \sin(t)\vec{i} + e^t \cos(t)\vec{j}$, $0 \le t \le \pi$.

A. $e^{3\pi} + 1$

B. $-e^{3\pi} - 1$

C. 0

D. $-\pi^3$

E. π^3

hard way: the same way as previous problem

find $\vec{r}\,'$, the rewrite $\vec{F}$ in terms of x, y from $\vec{r}$

then do $\int_C \vec{F} \cdot d\vec{r}$

easier way: we know $\vec{F}$ is conservative because we know

$\vec{F} = \vec{\nabla} \phi$

we can use the Fundamental Theorem of Line Integrals

$$\int_C \vec{\nabla}\phi \cdot d\vec{r} = \phi(B) - \phi(A)$$

end location

start location

find ϕ: $\vec{F} = \vec{\nabla}\phi$

$$\langle 3+2xy,\ x^2 - 3y^2 \rangle = \langle \phi_x,\ \phi_y \rangle$$

$$\phi_x = 3 + 2xy \quad (1)$$

$$\phi_y = x^2 - 3y^2 \quad (2)$$

integrate ① with respect to x

$$\phi = \int 3+2xy \, dx = 3x + x^2 y + \underbrace{a(y)}_{\text{function of } y \text{ only (or constant)}}$$

now take partial with respect to y and compare to ②

$$\phi_y = x^2 + \frac{da}{dy} = x^2 - 3y^2 \quad \text{from ②}$$

$$\frac{da}{dy} = -3y^2 \qquad \text{so,} \quad a = -y^3 + C$$

therefore, $\boxed{\phi = 3x + x^2 y - y^3 + C}$

$\vec{r}(t) = \langle e^t \sin t, e^t \cos t \rangle \quad 0 \le t \le \pi$

$B = \langle 0, -e^{\pi} \rangle$

$A = \langle 0, 1 \rangle$

$$\int_C \vec{\nabla}\phi \cdot d\vec{r} = \phi(B) - \phi(A) = \left[3(0) + (0)^2(-e^{\pi}) - (-e^{\pi})^3 + C\right] - \left[3(0) + (0)^2(1) - (1)^3 + C\right]$$

$$= e^{3\pi} + C - (-1 + C) = \boxed{e^{3\pi} + 1}$$

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check conservative : $\vec{F} = \langle f, g \rangle$

$\vec{F} = \vec{\nabla}\phi$ if $g_x = f_y$

$\vec{F} = \langle f, g, h \rangle$

$\vec{F} = \vec{\nabla}\phi$ if $g_x = f_y$

$h_x = f_z$

$g_z = h_y$