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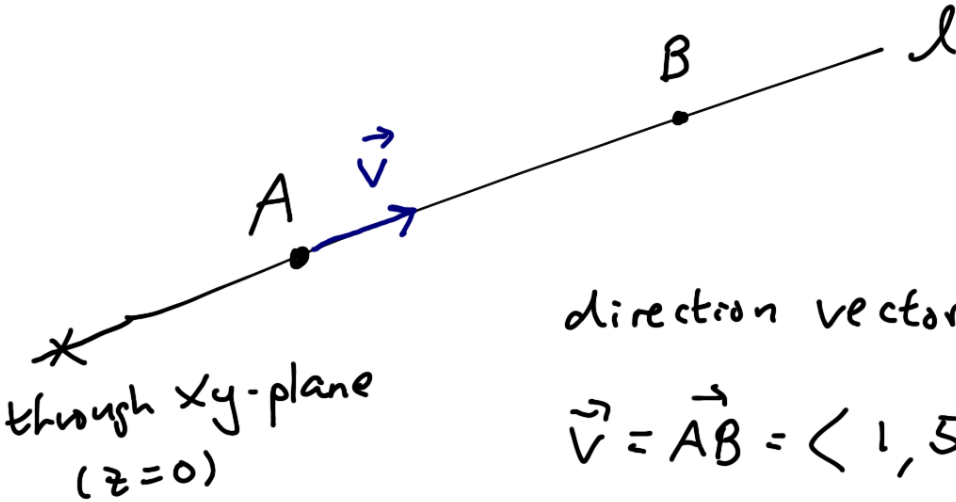
A line l passes through the points $A(1,-2,1)$ and $B(2,3,-1)$. At what point does this line intersect with the xy-plane?

A. $(\frac{3}{2}, \frac{-1}{2}, 0)$
B. $(\frac{5}{2}, \frac{-1}{2}, 0)$
C. $(\frac{3}{2}, -1, 0)$
D. $(\frac{5}{2}, \frac{1}{2}, 0)$
E. $(\frac{3}{2}, \frac{1}{2}, 0)$



direction vector $\vec{v}$: $\vec{AB}$ or $\vec{BA}$

$\vec{v} = \vec{AB} = \langle 1, 5, -2 \rangle$

line : $\vec{r}(t) = \vec{r}_0 + t\vec{v}$

vector to one point on line

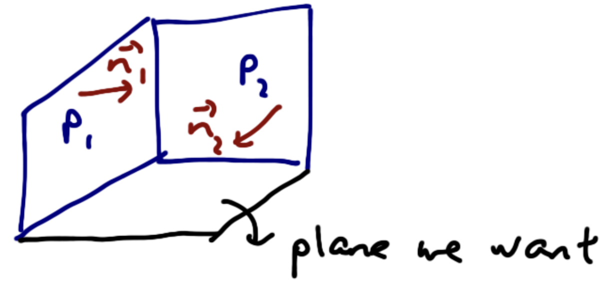
$\vec{r}(t) = \langle 1, -2, 1 \rangle + t \langle 1, 5, -2 \rangle$
$= \langle 1+t, -2+5t, 1-2t \rangle$

on xy-plane : $z = 0$
so $1 - 2t = 0$
$t = \frac{1}{2}$

point on xy-plane : $t = 1/2$

$\boxed{(3/2, 1/2, 0)}$

Find the equation of the plane that passes through the point (1, 1, −2) and is perpendicular to both the planes $\underbrace{2x + 2y - z = 1}_{P_1}$ and $\underbrace{x + 3z = 2}_{P_2}$.



A. $6x - 7y - 2z = 3$

B. $6x + 7y - z = 15$

C. $3x - y + z = 0$

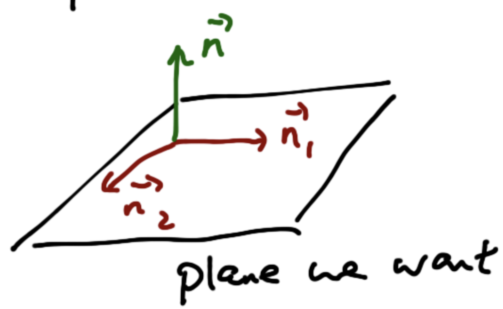
D. $6x - 8y - 2z = 2$

E. $3x - y + 2z = -2$

P_1: normal vector $\vec{n}_1 = \langle 2, 2, -1 \rangle$

P_2: normal vector $\vec{n}_2 = \langle 1, 0, 3 \rangle$

plane we want : need normal vector and a point



normal vector : $\boxed{\vec{n} = \vec{n}_1 \times \vec{n}_2}$ or $\vec{n}_2 \times \vec{n}_1$

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 2 & -1 \\ 1 & 0 & 3 \end{vmatrix} = \langle 6, -7, -2 \rangle$$

equation : $\vec{n} \cdot \langle x - x_0, y - y_0, z - z_0 \rangle = 0$

$\langle 6, -7, -2 \rangle \cdot (x - 1, y - 1, z + 2) = 0$

$6(x-1) - 7(y-1) - 2(z+2) = 0$ $\boxed{6x - 7y - 2z = 3}$

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Identify the surface defined by the equation $x^2 - y^2 + \underline{2z - z^2} = 2$.

complete the square

no hyperbola traces

A. ~~Elliptic paraboloid~~

B. Hyperboloid of one sheet

C. Hyperboloid of two sheets

D. ~~Ellipsoid~~ no hyperbola traces

E. ~~Hyperbolic paraboloid~~

no parabola traces

$$x^2 - y^2 - z^2 + 2z = 2$$

$$x^2 - y^2 - (z^2 - 2z) = 2$$

$$x^2 - y^2 - (z^2 - 2z + 1) = 2 - 1$$

actually -1

$$x^2 - y^2 - (z-1)^2 = 1$$

at $x = 0$: yz-trace $-y^2 - (z-1)^2 = 1$

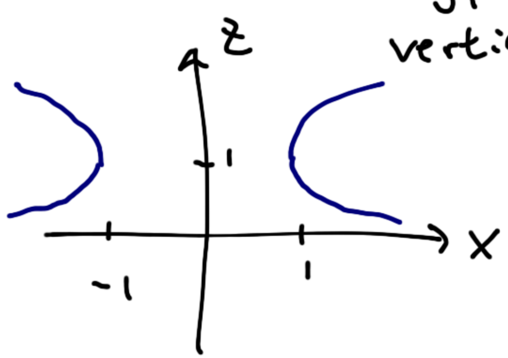
impossible → no yz-trace

shape does not cross yz-plane

at $y = 0$: xz-trace $x^2 - (z-1)^2 = 1$

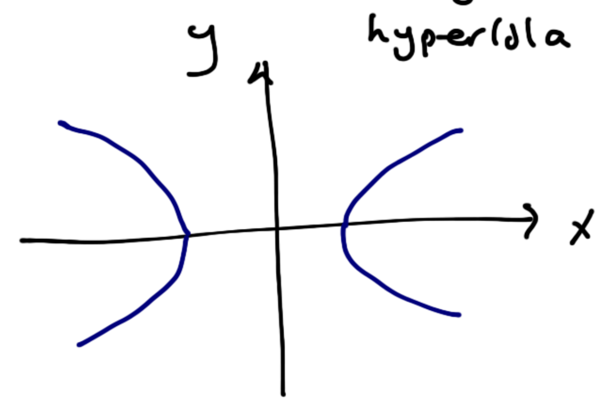
hyperbola

vertices on x-axis



at $z = 1$ $x^2 - y^2 = 1$

hyperbola



(at $z = 1$)

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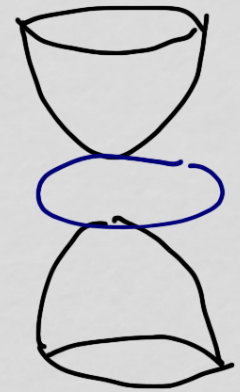
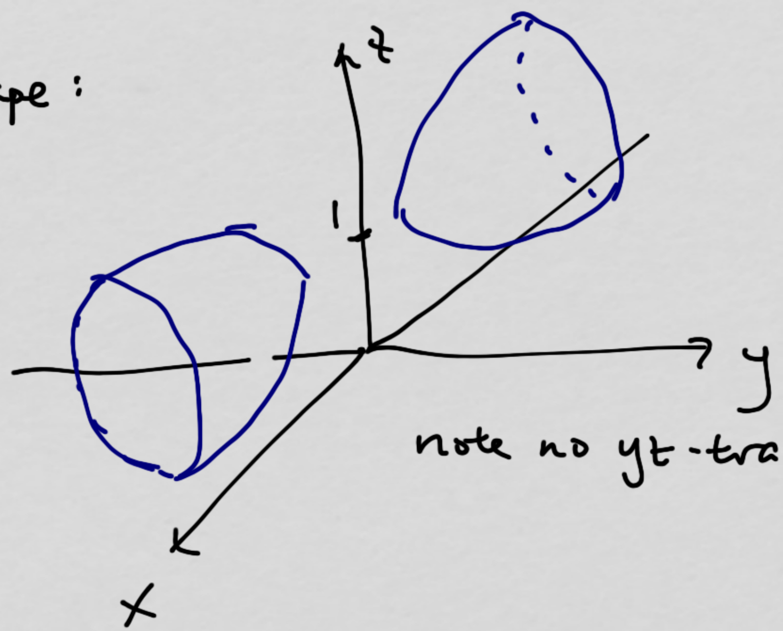
hyperboloid one sheet

hyperboloid two sheets



this matches what we see in our problem

our shape:



note no yz-trace

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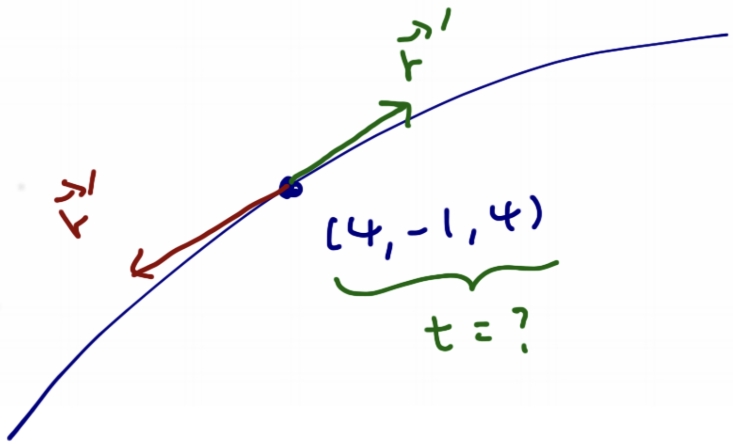
One vector parallel to the tangent to the curve

$$x = 4\sqrt{t},\ y = t^2 - 2,\ z = \frac{4}{t}$$

at the point $(4, -1, 4)$ is

$\vec{r}(t) = \langle 4\sqrt{t}, t^2-2, \frac{4}{t} \rangle$



A. $4\mathbf{i} - 9\mathbf{j} + 4\mathbf{k}$

B. $8\mathbf{i} + 6\mathbf{j} + \mathbf{k}$

C. $2\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$

D. $4\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$

E. $3\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}$

$\vec{r}(t) = \langle 4\sqrt{t}, t^2-2, \frac{4}{t} \rangle$

$\vec{r}\,'(t) = \langle \frac{2}{\sqrt{t}}, 2t, -\frac{4}{t^2} \rangle$

$\vec{r}\,'(1) = \langle 2, 2, -4 \rangle$

if $\vec{r}(t) = \langle 4, -1, 4 \rangle$

then
$$\left.\begin{aligned} 4\sqrt{t} &= 4 \\ t^2-2 &= -1 \\ \frac{4}{t} &= 4 \end{aligned}\right\}\ t = 1$$

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A particle is moving with acceleration $\mathbf{a} = \langle 0,\, 6t,\, 4\rangle$. If the position at time $t = 1$ is $\mathbf{r}(1) = \langle 0,\, 5,\, 1\rangle$ and the velocity at time $t = 0$ is $\mathbf{v}(0) = \langle -2,\, 2,\, -1\rangle$, then the position at time $t = 2$ is:

A. $\langle -1,\, 14,\, 2\rangle$

B. $\langle 1,\, -8,\, 12\rangle$

C. $\langle 3,\, -4,\, 5\rangle$

D. $\langle -2,\, 14,\, 6\rangle$

E. $\langle 1,\, 1,\, 2\rangle$

$\vec{a} = \langle 0, 6t, 4\rangle$

$\vec{v}(t) = \int \vec{a}\,dt = \langle c_1, 3t^2 + c_2, 4t + c_3\rangle$

we know $\vec{v}(0) = \langle -2, 2, -1\rangle = \langle c_1, c_2, c_3\rangle$

so, $\vec{v}(t) = \langle -2, 3t^2+2, 4t-1\rangle$

$\vec{r}(t) = \int \vec{v}(t)\,dt = \langle -2t + d_1, t^3 + 2t + d_2, 2t^2 - t + d_3\rangle$

we know $\vec{r}(1) = \langle 0, 5, 1\rangle = \langle -2 + d_1, 3 + d_2, 1 + d_3\rangle$

so $d_1 = 2$, $d_2 = 2$, $d_3 = 0$

so $\vec{r}(t) = \langle -2t+2, t^3+2t+2, 2t^2 - t\rangle$

$\vec{r}(2) = \langle -2, 14, 6\rangle$

The arclength of the curve $\vec{r}(t) = 2t\vec{i} + t^2\vec{j} + (\ln t)\vec{k}$ for $2 \le t \le 4$ is:

A. $\frac{17}{4}$

B. $4 + \ln 2$

C. $16 + \ln 2$

D. $\frac{15}{4}$

E. $12 + \ln 2$

$$\boxed{\text{length: } L = \int_a^b |\vec{r}\,'(t)|\,dt}$$

$$\vec{r}\,' = \langle 2, 2t, \tfrac{1}{t} \rangle$$

$$|\vec{r}\,'| = \sqrt{4 + 4t^2 + \frac{1}{t^2}} = \sqrt{\frac{4t^2 + 4t^4 + 1}{t^2}}$$

$$= \sqrt{\frac{(2t^2+1)^2}{t^2}} = \frac{2t^2+1}{t} = 2t + \frac{1}{t}$$

$$L = \int_2^4 \left(2t + \frac{1}{t}\right) dt = t^2 + \ln t \Big|_2^4 = 16 + \ln 4 - 4 - \ln 2$$

$$= 12 + \ln 4 - \ln 2$$

$$= 12 + 2\ln 2 - \ln 2 = 12 + \ln 2$$

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Evaluate

$$\lim_{(x,y)\to(0,0)} \frac{y^4 - x^4}{x^2 + y^2}\, e^{x^2+y^2}$$

A. 0

B. 1

C. -1

D. e^2

E. The limit does not exist

what if we plug in $(x,y) = (0,0)$?

$$\frac{0-0}{0+0}\, e^0 \quad \text{undefined}$$

must take into account of all possible paths

or find a path-independent way

$$\lim_{(x,y)\to(0,0)} \frac{y^4 - x^4}{x^2+y^2}\, e^{x^2+y^2}$$

$$= \lim_{(x,y)\to(0,0)} \frac{(y^2-x^2)\cancel{(y^2+x^2)}}{\cancel{x^2+y^2}}\, e^{x^2+y^2}$$

$$= \lim_{(x,y)\to(0,0)} (y^2-x^2)\, e^{x^2+y^2} = 0 \cdot e^0 = 0$$

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Find the directional derivative of the function $f(x, y, z) = x^2y + y^2z$ at $(1, 2, 3)$ in the direction toward the point $(3, 1, 5)$.

$$D_{\vec{u}} f(x,y,z) = \vec{\nabla} f(x,y,z) \cdot \vec{u}$$

unit vector specifying direction

gradient

A. 1

B. 3

C. $\frac{1}{3}$

D. -2

E. -1

$$\vec{\nabla} f = \langle f_x, f_y, f_z \rangle = \langle 2xy, x^2+2yz, y^2 \rangle$$

at $(1,2,3)$, $\vec{\nabla} f = \langle 4, 13, 4 \rangle$

now $\vec{u}$: vector from $(1,2,3)$ to $(3,1,5)$ as a unit vector

$$\vec{u} = \frac{\langle 2, -1, 2 \rangle}{|\langle 2, -1, 2 \rangle|} = \frac{\langle 2, -1, 2 \rangle}{\sqrt{9}} = \left\langle \frac{2}{3}, -\frac{1}{3}, \frac{2}{3} \right\rangle$$

$$D_{\vec{u}} f = \langle 4, 13, 4 \rangle \cdot \left\langle \frac{2}{3}, -\frac{1}{3}, \frac{2}{3} \right\rangle = \frac{8}{3} - \frac{13}{3} + \frac{8}{3} = 1$$

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Which of the following is an equation for the plane tangent to the surface

$$z = (x^2 + y^2)^{1/3} \text{ at the point } (2, 2, 2)?$$

let $F = (x^2+y^2)^{1/3} - z$ so $z = (x^2+y^2)^{1/3}$ is a level surface of F

and we know $\vec{\nabla}F$ is normal to a level surface/curve and thus can be used as the normal vector of the tangent plane

A. $z = x + y - 2$

B. $3z = x + y + 2$

C. $3z = x + 2y$

D. $4z = 6x + 8y - 20$

E. $4z = x + y + 4$

$$\vec{\nabla}F = \left\langle \frac{1}{3}(x^2+y^2)^{-2/3}(2x),\ \frac{1}{3}(x^2+y^2)^{-2/3}(2y),\ -1 \right\rangle$$

$$= \left\langle \frac{2x}{3(x^2+y^2)^{2/3}},\ \frac{2y}{3(x^2+y^2)^{2/3}},\ -1 \right\rangle$$

at $(2,2,2)$

$$\vec{\nabla}F = \left\langle \frac{1}{3},\ \frac{1}{3},\ -1 \right\rangle$$ $\to$ can use as $\vec{n}$ for our tangent plane

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plane w/ $\vec{n} = \langle \frac{1}{3}, \frac{1}{3}, -1 \rangle$ and point $(2, 2, 2)$

$$\frac{1}{3}(x-2) + \frac{1}{3}(y-2) - 1\cdot(z-2) = 0$$

$$(x-2) + (y-2) - 3(z-2) = 0$$

$$x + y - 3z = -2$$

$$3z = x + y + 2$$