6.3 Ecological Models : Predators and Competitors (part 1)

predator : thing that eats prey (e.g. hawks)

prey : thing that predator eats (e.g. squirrels)

predator-prey system: x : prey $\quad y$: predator

$$\frac{dx}{dt} = ax - pxy = (a - py)x$$

$$\frac{dy}{dt} = -by + gxy = (-b + gx)y$$

a, b, p, g <u>positive</u> constants

the xy terms are interaction terms → how species interact w/ each other

prey

if $p = g = 0$, $\quad \frac{dx}{dt} = ax \quad$ → simple exponential growth

predator

$\frac{dy}{dt} = -by \quad$ → " " decay

if $p \cdot g \neq 0$ $\quad \frac{dx}{dt} = (\underbrace{a - py}_{\text{rate of growth decreased by } y})x$

$\frac{dy}{dt} = (-b + gx)y$ rate increased by x

<u>example</u> $\frac{dx}{dt} = x - 0.5xy = f$ $\qquad x$: prey

y: predator

$\frac{dy}{dt} = -0.75y + 0.25xy = g$

cp : $x' = y' = 0 \to$ pop. not changing

$x - 0.5xy = 0 \to x(1 - 0.5y) = 0$

$\boxed{x = 0}$, $\boxed{y = 2}$ $\qquad$ both eqs. $= 0$

$-0.75y + 0.25xy \to 0.25y(-3 + x) = 0$

$\boxed{y = 0}$, $\boxed{x = 3}$

$(0, 0)$, $(3, 2)$ coexistence

$\downarrow$

both die out

which is stable?

linearize about each cp

Jacobian : $J = \begin{bmatrix} f_x & f_y \\ g_x & g_y \end{bmatrix} = \begin{bmatrix} 1-0.5y & -0.5x \\ 0.25y & -0.75+0.25x \end{bmatrix}$

near each cp: (0,0), (3,2)

linear sys: $\begin{bmatrix} u' \\ v' \end{bmatrix} = J \begin{bmatrix} u \\ v \end{bmatrix}$ $\quad u = x - x_0$, $v = y - y_0$ (x,y) of cp

near (0,0)

$$\begin{bmatrix} u' \\ v' \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -0.75 \end{bmatrix}}_{\lambda = 1,\, -0.75} \begin{bmatrix} u \\ v \end{bmatrix}$$

saddle pt, unstable

unless start w/ x=0, y=0

t→0 will NOT go here

(they don't die out)

near (3, 2)

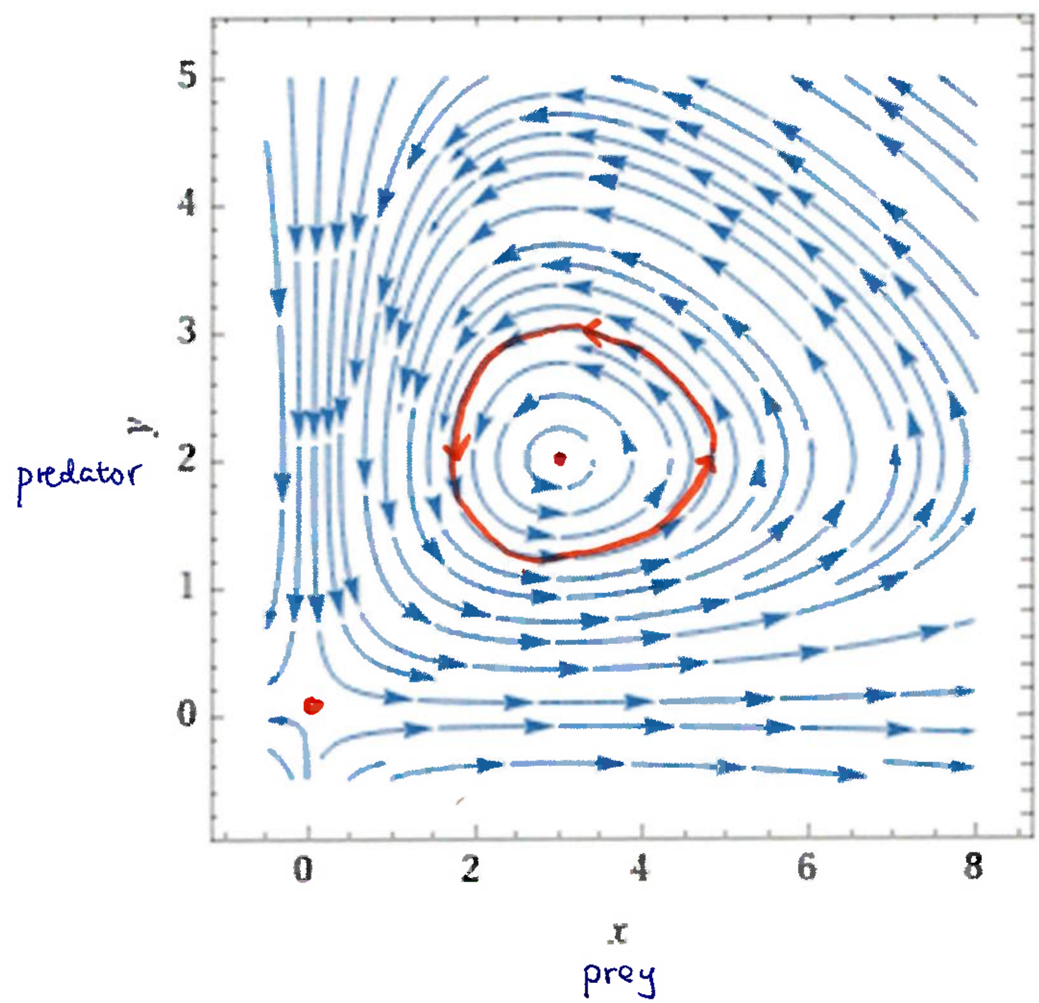
$$\begin{bmatrix} u' \\ v' \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -1.5 \\ 0.5 & 0 \end{bmatrix}}_{\lambda \,:\, \text{purely imaginary}} \begin{bmatrix} u \\ v \end{bmatrix}$$

center, stable

BUT, remember this is a case where the linearized sys may "lie"

$$\frac{dx}{dt} = x - 0.5xy$$

$$\frac{dy}{dt} = -0.75y + 0.25xy$$



<u>example</u>

$$\frac{dx}{dt} = x - 0.5x^2 - 0.5xy$$

$$\frac{dy}{dt} = -0.25y + 0.5xy$$

w/o interaction: $\frac{dx}{dt} = x - 0.5x^2 = x\underbrace{(1-0.5x)}_{\text{if } x=2,\ x'=0}$

prey pop. capped at 2

$\frac{dy}{dt} = -0.25y$ just decays

cp: $(0,0)$, $(2,0)$, $(1/2, 3/2)$

$$J = \begin{bmatrix} 1 - x - 0.5y & -0.5x \\ 0.5y & -0.25 + 0.5x \end{bmatrix}$$

at $(0,0)$ $J = \begin{bmatrix} 1 & 0 \\ 0 & -0.25 \end{bmatrix}$ saddle pt, unstable

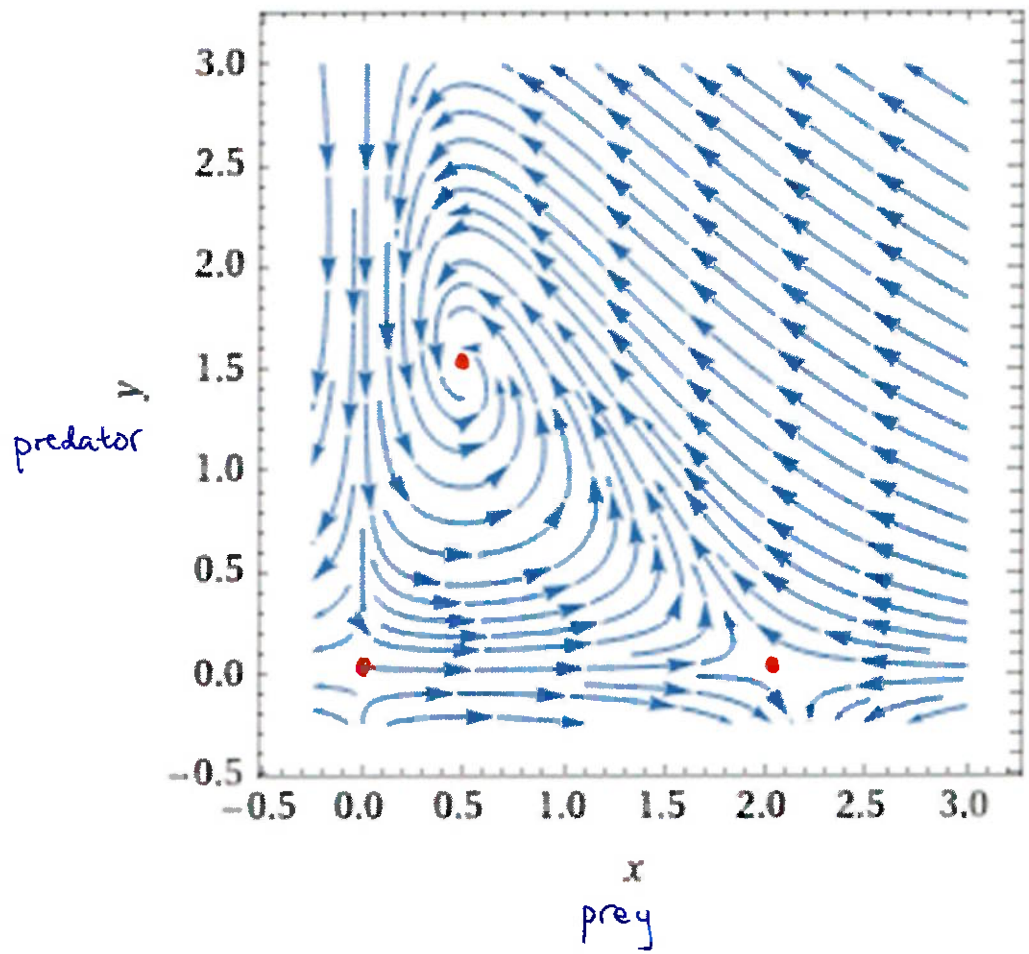
at $(2,0)$ $J = \begin{bmatrix} -1 & -1 \\ 0 & 0.75 \end{bmatrix}$ saddle pt, unstable

at $(1/2, 3/2)$ $J = \begin{bmatrix} -0.25 & -0.25 \\ 0.75 & 0 \end{bmatrix}$ $\lambda = -0.125 \pm 0.415i$

AS spiral

$$\frac{dx}{dt} = x - 0.5x^2 - 0.5xy$$

$$\frac{dy}{dt} = -0.25y + 0.5xy$$



example

$$\frac{dx}{dt} = -x + \frac{1}{2}x^2 - \frac{1}{2}xy$$

$$\frac{dy}{dt} = y^2 - 4y + xy$$

---

example

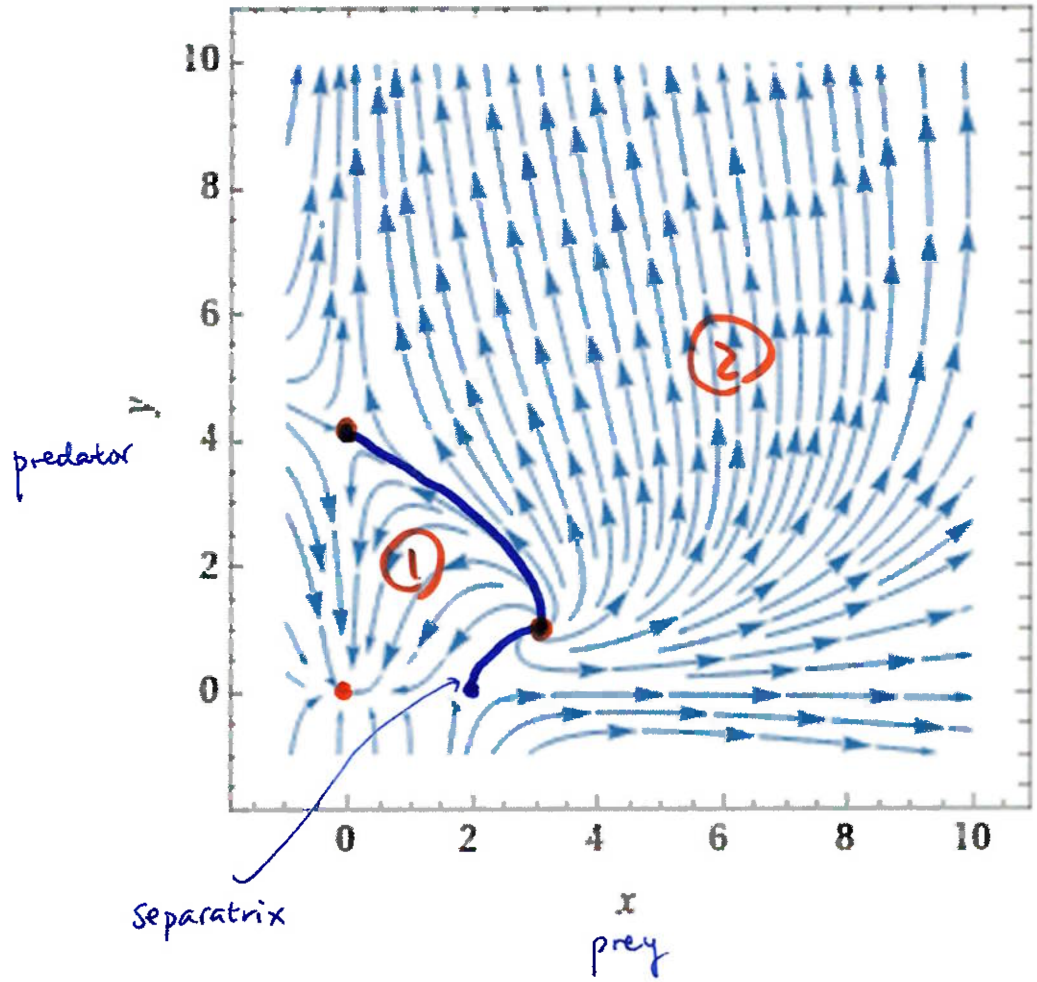
$$\frac{dx}{dt} = 14x - \frac{1}{2}x^2 - xy$$

$$\frac{dy}{dt} = 16y - \frac{1}{2}y^2 - xy$$

next time: competition system

$$\frac{dx}{dt} = -x + \frac{1}{2}x^2 - \frac{1}{2}xy$$

$$\frac{dy}{dt} = y^2 - 4y + xy$$



1 : both die out

2 : pops. explode

$$\frac{dx}{dt} = 14x - \frac{1}{2}x^2 - xy$$

$$\frac{dy}{dt} = 16y - \frac{1}{2}y^2 - xy$$