6.3 Predators and Competitors (part 2)

competitors: two species going after the same food source but otherwise leave each other alone (e.g. chipmunks and squirrels)

$$\frac{dx}{dt} = a_1 x - b_1 x^2 - c_1 xy = x(a_1 - b_1 x - c_1 y)$$

$$\frac{dy}{dt} = a_2 y - b_2 y^2 - c_2 xy = y(a_2 - b_2 y - c_2 x)$$

$a_i, b_i, c_i > 0$

what do these mean?

if $c_i = 0$ (no interaction)

$$\frac{dx}{dt} = x(a_1 - b_1 x)$$ pop. capped $x = \frac{a_1}{b_1}$

$$\frac{dy}{dt} = y(a_2 - b_2 y)$$ pop. capped at $y = \frac{a_2}{b_2}$

if $c_i \neq 0$ (>0) $\quad \frac{dx}{dt} = x\,(\underbrace{a_1 - b_1 x}_{\text{logistic}} - \underbrace{c_1 y}_{\text{serves as a reduction to } x'})$

cp of the system: $\underbrace{(0,0)}_{\substack{\text{both}\\\text{die}}},\ \underbrace{(0,\ a_2/b_2)}_{\substack{y \text{ survives}\\\text{at cap}}},\ \underbrace{(a_1/b_1,\ 0)}_{\substack{x \text{ survives}\\\text{at cap}}},\ \underbrace{(x_c,\ y_c)}_{\text{coexistence}}$

$$\frac{dx}{dt} = x\,(a_1 - b_1 x - c_1 y) = x\left[a_1 - (b_1 x + c_1 y)\right]$$

$$\frac{dy}{dt} = y\,(a_2 - b_2 y - c_2 x) = y\left[a_2 - (b_2 y + c_2 x)\right]$$

if $\frac{b_1}{c_1} > 1 \rightarrow$ x constrained more by its pop. cap than interaction

(similarly for y $\frac{b_2}{c_2} > 1$)

$\Rightarrow \frac{b_1}{c_1} \cdot \frac{b_2}{c_2} > 1 \rightarrow \boxed{b_1 b_2 > c_1 c_2}$ the limitation of own species more important than the other

$\boxed{\text{coexistence}}$

we can do similar analysis to show that

if $\boxed{b_1 b_2 < c_1 c_2}$ other species' presence dominates

one species goes extinct

example

$$\frac{dx}{dt} = 60x - 3x^2 - 4xy \qquad b_1 = 3 \quad c_1 = 4$$

$$\frac{dy}{dt} = 42y - 3y^2 - 2xy \qquad b_2 = 3 \quad c_2 = 2$$

$b_1 b_2 = 9 \quad c_1 c_2 = 8$

should lead to coexistence

cp: $(0,0), (0,14), (20,0), (12,6)$

linearize about each cp to determine stability

$$J = \begin{bmatrix} 60 - 6x - 4y & -4x \\ -2y & 42 - 6y - 2x \end{bmatrix}$$

at $(0,0)$

$$J = \begin{bmatrix} 60 & 0 \\ 0 & 42 \end{bmatrix} \qquad \lambda = 60, 42$$

unstable node

at $(0,14)$ $\quad J = \begin{bmatrix} 4 & 0 \\ -28 & -42 \end{bmatrix}$ $\quad \lambda = 4, -42$

saddle pt, unstable

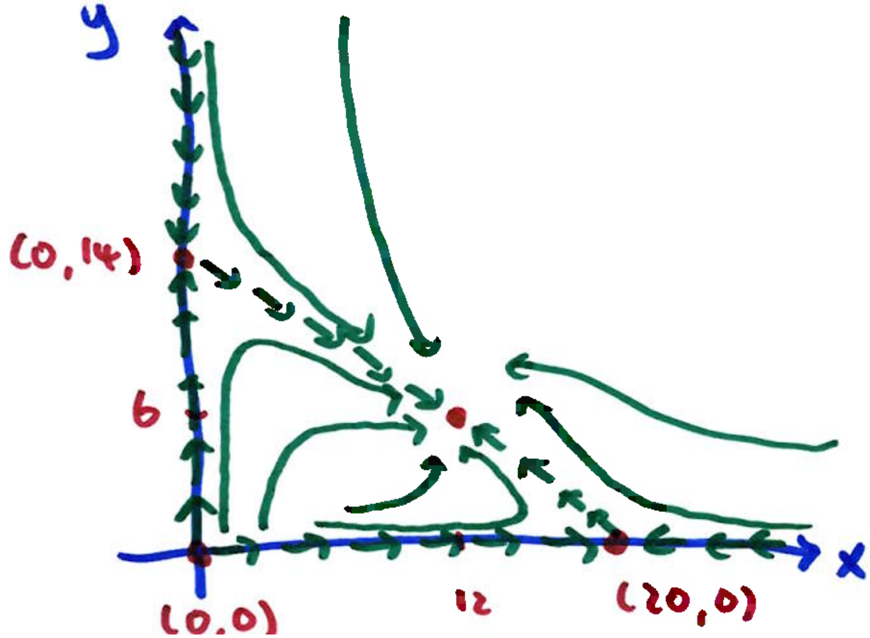
at $(20,0)$ $\quad J = \begin{bmatrix} -60 & -80 \\ 0 & 2 \end{bmatrix}$ $\quad \lambda = -60, 2$

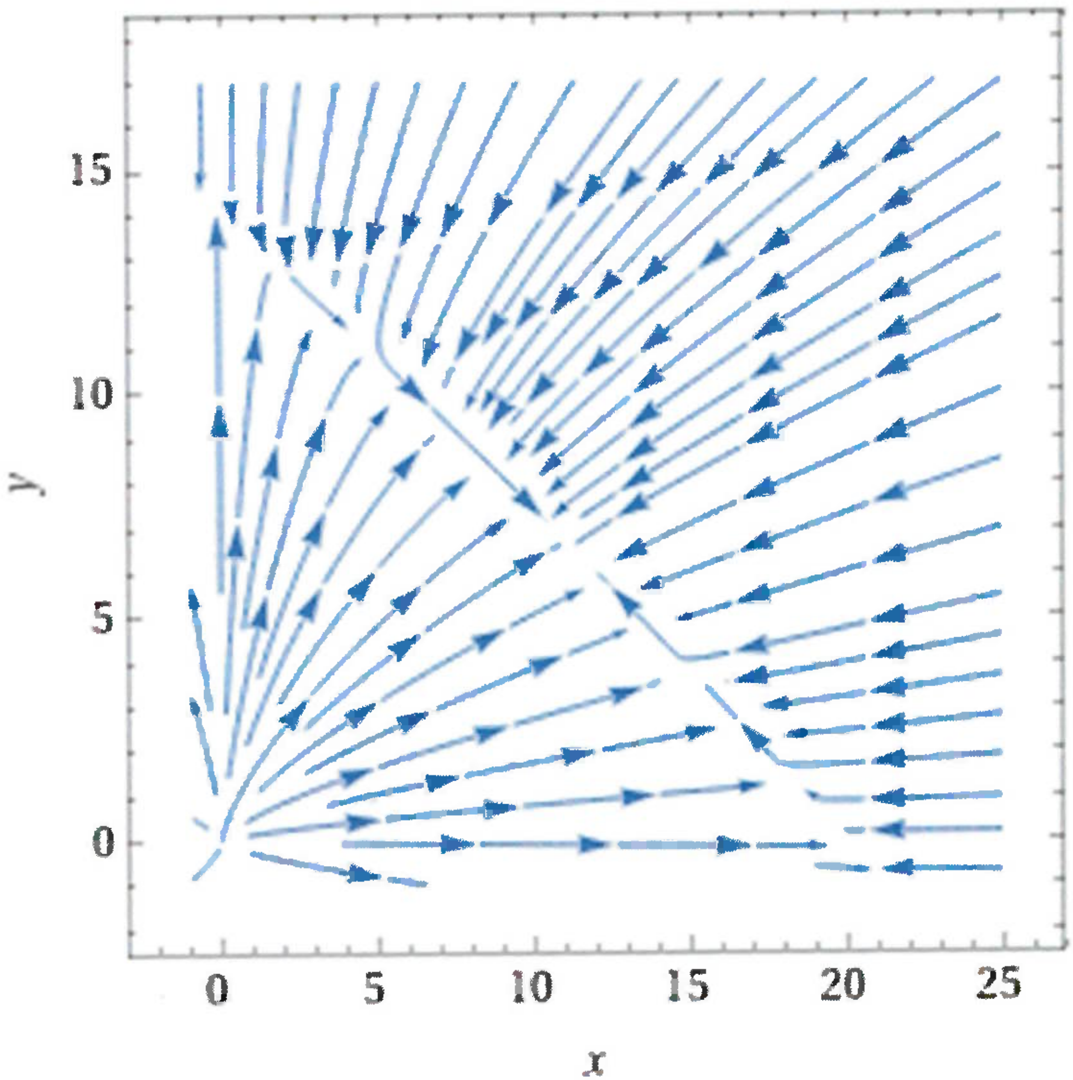
saddle, unstable

at $(12,6)$ $\quad J = \begin{bmatrix} -36 & -48 \\ -12 & -18 \end{bmatrix}$ $\quad \lambda$'s < 0 real

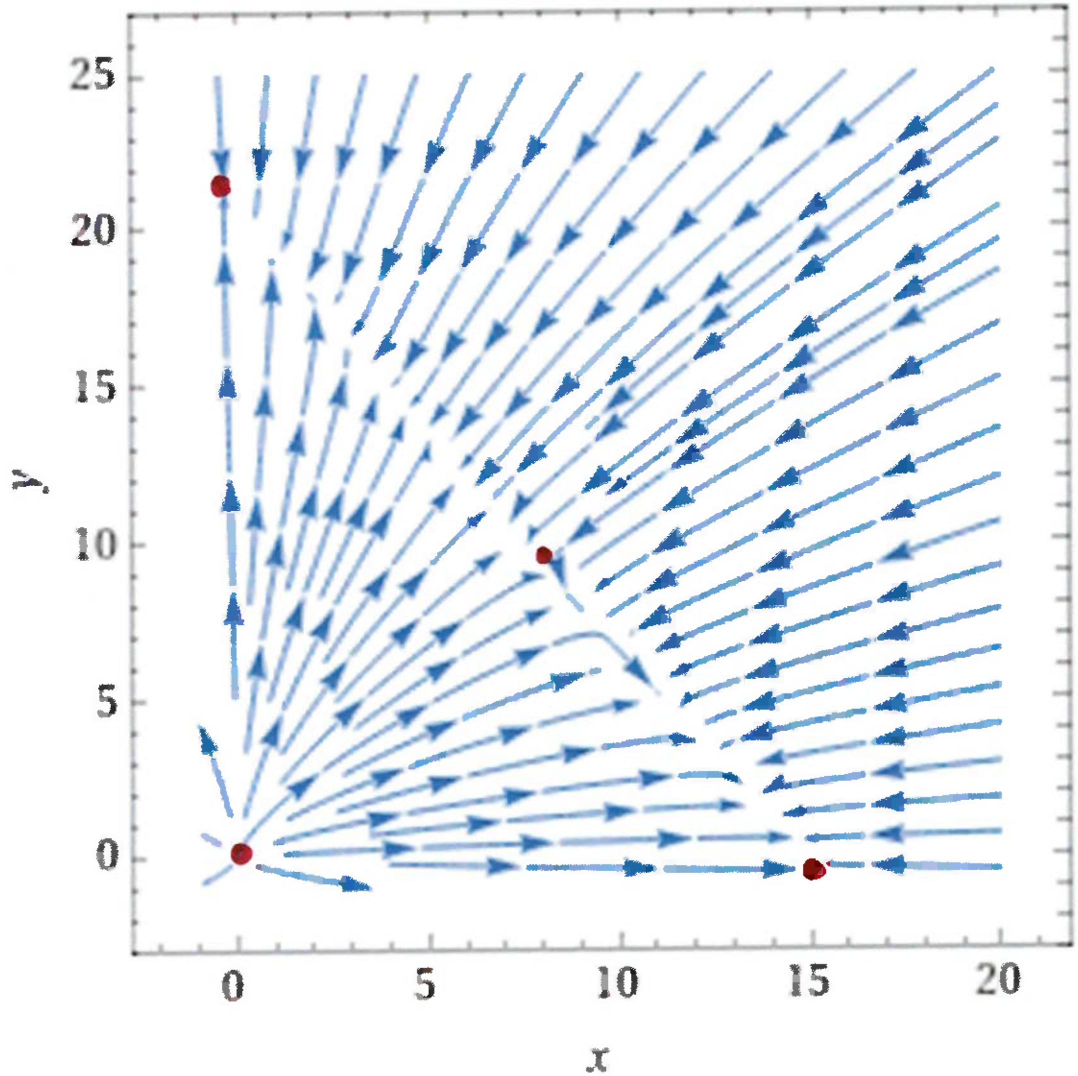
asymp. stable node

sketch of phase diagram





15
10
5
0
y
0
5
10
15
20
25
x



25
20
15
10
5
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y
0
5
10
15
20
x

example

$$\frac{dx}{dt} = 60x - 4x^2 - 3xy \qquad b_1 = 4 \qquad c_1 = 3$$

$$\frac{dy}{dt} = 42y - 2y^2 - 3xy \qquad b_2 = 2 \qquad c_2 = 3$$

$b_1 b_2 < c_1 c_2$ competition dominates

one species dies out

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back to $\frac{dx}{dt} = a_1 x - b_1 x^2 - c_1 xy = x(a_1 - b_1 x - c_1 y)$

positive

$$\frac{dy}{dt} = a_2 y - b_2 y^2 - c_2 xy = y(a_2 - b_2 y - c_2 x)$$

if $c_i < 0$ → boost to rate of growth → cooperation

(other species helps growth

e.g. ants and aphids)

if $c_i > 0$ → decrease in rate due to other → predation

$$\frac{dx}{dt} = 30x - 3x^2 + xy \qquad b_1 = 3 \qquad c_1 = -1$$

$$\frac{dy}{dt} = 60y - 3y^2 + 4xy \qquad b_2 = 3 \qquad c_2 = -4$$

$\Bigg\}$ cooperation