Complex eigenthings

$$\vec{x}' = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \vec{x}$$

solution: $\vec{x}(t) = \underbrace{c_1 e^{\lambda_1 t} \vec{v}_1}_{\vec{x}_1} + \underbrace{c_2 e^{\lambda_2 t} \vec{v}_2}_{\vec{x}_2}$

$\det(A - \lambda I) = 0$

$$\begin{vmatrix} 1-\lambda & 2 \\ -2 & 1-\lambda \end{vmatrix} = 0$$

$(1-\lambda)^2 + 4 = 0$ $\quad 1-\lambda = \pm 2i$

$\lambda_1 = 1+2i, \quad \lambda_2 = 1-2i$

$\lambda_1 = 1+2i$

$-2i \cdot i = -2i^2 = -2 \cdot -1 = 2$

$(A - \lambda_1 I)\vec{v}_1 = \vec{0}$

$$\left[\begin{array}{cc|c} -2i & 2 & 0 \\ -2 & -2i & 0 \end{array}\right] \underset{\substack{\text{mult. R1 by } i \\ \text{add to R2}}}{\sim} \left[\begin{array}{cc|c} -2i & 2 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} i & -1 & 0 \\ 0 & 0 & 0 \end{array}\right] \qquad \vec{v}_1 = \begin{bmatrix} a \\ b \end{bmatrix}$$

$b = r$

$R1: \; ia - b = 0$

$ia = b \qquad a = \frac{b}{i} = \frac{r}{i}$

$r = i \rightarrow a = 1, \; b = i$

$$\boxed{\vec{v}_1 = \begin{bmatrix} 1 \\ i \end{bmatrix} \qquad \lambda_1 = 1+2i}$$

$\underline{\lambda_2 = 1-2i}$

$(A - \lambda_2 I)\vec{v}_2 = \vec{0}$

$$\left[\begin{array}{cc|c} 2i & 2 & 0 \\ -2 & 2i & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 2i & 2 & 0 \\ 0 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} i & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$\boxed{\vec{v}_2 = \begin{bmatrix} 1 \\ -i \end{bmatrix} \qquad \lambda_2 = 1-2i}$$

$$\vec{x}_1 = e^{\lambda_1 t}\vec{v}_1 = e^{(1+2i)t}\begin{bmatrix} 1 \\ i \end{bmatrix} = e^t e^{i(2t)}\begin{bmatrix} 1 \\ i \end{bmatrix}$$

$$e^{ix} = \cos(x) + i\sin(x)$$

$$= e^t\left(\cos(2t) + i\sin(2t)\right)\begin{bmatrix} 1 \\ i \end{bmatrix}$$

$$= e^t\begin{bmatrix} \cos(2t) + i\sin(2t) \\ -\sin(2t) + i\cos(2t) \end{bmatrix} = \underbrace{\boxed{e^t\begin{bmatrix} \cos(2t) \\ -\sin(2t) \end{bmatrix} + i\,e^t\begin{bmatrix} \sin(2t) \\ \cos(2t) \end{bmatrix}}}_{\vec{x}_1}$$

$$\vec{x}_2 = e^{\lambda_2 t}\vec{v}_2 = e^{(1-2i)t}\begin{bmatrix} 1 \\ -i \end{bmatrix}$$

$$e^{i(-2t)} = \cos(-2t) + i\sin(-2t)$$
$$= \cos(2t) - i\sin(2t)$$

$$= e^t\left(\cos(2t) - i\sin(2t)\right)\begin{bmatrix} 1 \\ -i \end{bmatrix}$$

$$= e^t\begin{bmatrix} \cos(2t) - i\sin(2t) \\ -\sin(2t) - i\cos(2t) \end{bmatrix} = \underbrace{\boxed{e^t\begin{bmatrix} \cos(2t) \\ -\sin(2t) \end{bmatrix} - i\,e^t\begin{bmatrix} \sin(2t) \\ \cos(2t) \end{bmatrix}}}_{\vec{x}_2}$$

gen. solution: $\vec{x}(t) = \underbrace{A}_{} \vec{x}_1 + \underbrace{B}_{} \vec{x}_2$

complex pairs too

$$\vec{x}(t) = Ae^{t}\begin{bmatrix}\cos(2t)\\ -\sin(2t)\end{bmatrix} + iAe^{t}\begin{bmatrix}\sin(2t)\\ \cos(2t)\end{bmatrix}$$

$$+ Be^{t}\begin{bmatrix}\cos(2t)\\ -\sin(2t)\end{bmatrix} - iBe^{t}\begin{bmatrix}\sin(2t)\\ \cos(2t)\end{bmatrix}$$

$$= \underbrace{\overset{C_1}{(A+B)}}_{\text{real}} e^{t}\begin{bmatrix}\cos(2t)\\ -\sin(2t)\end{bmatrix} + \underbrace{\overbrace{i(A-B)}^{C_2}}_{\text{real}}\begin{bmatrix}\sin(2t)\\ \cos(2t)\end{bmatrix}$$

b/c complex conj. pairs

$$= C_1 e^{t}\begin{bmatrix}\cos(2t)\\ -\sin(2t)\end{bmatrix} + C_2 e^{t}\begin{bmatrix}\sin(2t)\\ \cos(2t)\end{bmatrix}$$

Application of stability

747 at 40,000 ft cruising altitude

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -0.003 & 0.039 & 0 & -0.322 \\ -0.065 & -0.319 & 7.74 & 0 \\ 0.020 & -0.101 & -0.429 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u+u_w \\ v+v_w \\ q \\ \theta \end{bmatrix} + \underbrace{\vec{w}}_{\text{control}}$$

$q = \dot{\theta}$

Solution: $\begin{bmatrix} u(t) \\ v(t) \\ q(t) \\ \theta(t) \end{bmatrix} = \cdots$ stable?

$$\lambda = \underbrace{\overbrace{-0.375}^{\text{strong damp}} \pm \overbrace{0.8818}^{\text{high freq.}}\, i}_{}$$

asympt. stable!

$$\lambda = \underbrace{\underbrace{-0.0005}_{\text{weak damping}} \pm \underbrace{0.0694}_{\text{low freq}}\, i}_{}$$

Short period oscillation

period ≈ 7 seconds

decays in ≈ 10 seconds

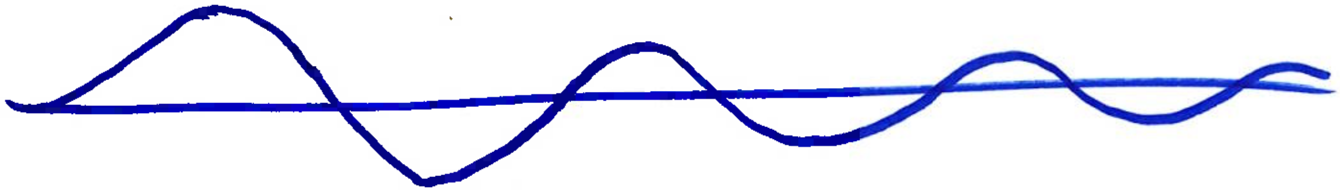
no effect on altitude and speed

phugoid oscillation

period 93 sec

decays in 80 min

affects speed and altitude



horiz.

Ch. 7 Laplace Transform

review: integration by parts → $\int f(x)dx = uv - \int v\,du$

improper integral → $\int_a^b f(x)dx$ $a, b = \pm\infty$
or if $f(x)$ DNE
$a < x < b$

$\int x e^x dx$ (Laplace: $\int_0^\infty f(t)\, e^{-st} dt$)

order of picking u: LIATE

- L og
- I nv. trig
- A lgebraic
- T rig
- E xp

$\int \underbrace{x}_{\text{alg.}} \underbrace{e^x}_{\text{exp.}} dx$

$u = x \qquad dv = e^x\,dx \qquad uv - \int v\,du = xe^x - \int e^x\,du$

$du = dx \qquad v = e^x \qquad\qquad = xe^x - e^x + C$

<u>Improper</u>

$$\int_0^\infty x e^{-x}\,dx = \lim_{a\to\infty} \int_0^a x e^{-x}\,dx$$

$u = x \qquad dv = e^{-x}\,dx$

$du = dx \qquad v = -e^{-x}$

$$= \lim_{a\to\infty} \left(-xe^{-x}\Big|_0^a + \int_0^a e^{-x}\,dx \right)$$

$$= \lim_{a\to\infty} \left(-ae^{-a} - e^{-x}\Big|_0^a \right)$$

$$= \lim_{a\to\infty} \left(-\cancelto{0}{ae^{-a}} - \cancelto{0}{e^{-a}} + 1 \right) = 1$$

is a number

→ integral converges