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SCATTERING IN QUANTUM FIELD THEORIES WITH SUPERGROUP INVARIANCE

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Abstract We conjecture the factorized scattering description for $OSP(m/2n)/OSP(m-1/2n)$ supersphere sigma models and $OSP(m/2n)$ Gross-Neveu models. The non unitarity of these field theories, which translates into a lack of "physical unitarity" of the S matrices, is a sticky issue, but we find that formal thermodynamic Bethe ansatz calculations appear meaningful, reproduce the correct central charges, and agree with perturbative calculations. This hopefully paves the way to a more thorough study of these and other models with supergroup symmetries using the S matrix approach.

Keywords: $OSP(m/2n)$ field theories, scattering theory, supergroups

1. Introduction

The field theory approach to phase transitions in disordered systems has realized major progress over the last few years, thanks to an ever deeper understanding of two dimensional field theories. Conformal invariance, combined with elegant reformulations using supersymmetry [1, 2, 22], and a greater control of non unitarity issues [4, 5, 14], now severely constrains the possible fixed points [7, 8].

Remarkably, the chief non perturbative method, the integrable approach, has not been pushed very far to study these models. This is a priori surprising. For instance, several disordered problems involve variants of the $OSP(m/2n)$ Gross-Neveu model, which formally appears just as integrable as its well known $O(N)$ counterpart. The standard way of proceeding to study such a model would be to determine its S matrix, and then use the thermodynamic Bethe ansatz and form-factors to calculate physical properties. This approach was pioneered in the elegant papers [9, 10], and revived in [11], but so far the

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subject was only touched upon in our opinion; for instance, although the S matrix of the $OSP(2/2)$ Gross-Neveu model has been conjectured [11], no calculation to justify this conjecture has been possible. Super sigma models have also been tackled, this time in the context of string theory [12], but there again results have only been very partial, and the S matrix approach even less developed than for super Gross-Neveu models.

The main reasons for this unsatisfactory situation seem technical. While there has been tremendous progress in understanding the sine-Gordon model and the $O(3)$ sigma models - the archetypes of integrable field theories - models based on other Lie algebras are only partially understood (see [13, 14] for some recent progress), and the situation becomes even more confusing when it comes to super-algebras. One of the main difficulties in understanding these theories is physical, and related with a general lack of unitarity - a feature that is natural from the disordered condensed matter point of view, but confusing at best from the field theory stand point. Another difficulty is simply the complexity of the Bethe ansatz for higher rank algebras, in particular super algebras.

The present note is a short summary of our ongoing work on the integrable approach for the case of $OSP(m/2n)$ field theories. We will discuss briefly two kinds of models, the supersphere sigma-models, and the Gross-Neveu models, mostly for algebras $OSP(1/2n)$. In each case, we will conjecture a scattering theory, whose striking feature will be the lack of unitarity of the S matrices, as a result of the supergroup symmetry. We will argue that formal thermodynamic calculations do make sense nevertheless, and illustrate this point for both types of models. More details will appear in [15].

2. Algebraic generalities

There are two basic integrable models with $O(N)$ symmetry, the Gross-Neveu model and the sphere sigma model $S^{N-1} = O(N)/O(N-1)$. The scattering theory for the $O(2P+1)$ Gross-Neveu model was completed only very recently [16]. However, the scattering of particles in the defining representation has been known for a long time [17] for both models, and this is where we would like to start here.

Scattering matrices with $O(N)$ symmetry can generally be written in terms of three independent tensors:

$$\check{S}_{i_1 j_1}^{j_2 i_2} = \sigma_1 E + \sigma_2 P + \sigma_3 I \,, \tag{1}$$

where we have set,

$$E_{i_1 j_1}^{j_2 i_2} = \delta_{i_1 j_1} \delta^{i_2 j_2} \,, \qquad P_{i_1 j_1}^{j_2 i_2} = \delta_{i_1}^{i_2} \delta_{j_1}^{j_2} \,, \qquad I_{i_1 j_1}^{j_2 i_2} = \delta_{i_1}^{j_2} \delta_{i_2}^{j_1} \,. \tag{2}$$

We are interested here in models for which none of the amplitudes vanish. Specifically, for N a positive integer, there are generically two known models whose scattering matrix for the vector representation has the form (1), with none of the σ_i's vanishing. They are given by

$$\sigma_1 = -\frac{2i\pi}{(N-2)(i\pi-\theta)}\,\sigma_2\,, \qquad \sigma_3 = -\frac{2i\pi}{(N-2)\theta}\,\sigma_2\,, \tag{3}$$

with two possible choices for σ_2:

$$\sigma_2^{\pm}(\theta) = \frac{\Gamma\left(1-\frac{\theta}{2i\pi}\right)\Gamma\left(\frac{1}{2}+\frac{\theta}{2i\pi}\right)}{\Gamma\left(\frac{\theta}{2i\pi}\right)\Gamma\left(\frac{1}{2}-\frac{\theta}{2i\pi}\right)}\frac{\Gamma\left(\pm\frac{1}{N-2}+\frac{\theta}{2i\pi}\right)}{\Gamma\left(1\pm\frac{1}{N-2}-\frac{\theta}{2i\pi}\right)}\frac{\Gamma\left(\frac{1}{2}\pm\frac{1}{N-2}-\frac{\theta}{2i\pi}\right)}{\Gamma\left(\frac{1}{2}\pm\frac{1}{N-2}+\frac{\theta}{2i\pi}\right)}. \tag{4}$$

The factor σ_2^+ does not have poles in the physical strip for $N \geq 0$, and the corresponding S matrix for $N \geq 3$ is believed to describe the $O(N)/O(N-1)$ sphere (S^{N-1}) sigma model. The factor σ_2^- does not have poles in the physical strip for $N \leq 4$. For $N > 4$, it describes the scattering of vector particles in $O(N)$ Gross-Neveu model. Recall that for $N = 3, 4$ the vector particles are unstable and disappear from the spectrum, that contains only kinks.

Our next step is to try to define models for which $N < 1$, in particular $N = 0$, or N negative. A similar question has been tackled by Zamolodchikov [18] under the condition that particles be "impenetrable", that is $\sigma_1 = 0$. The (standard) procedure he used was to study the algebraic relations satisfied by the objects E, I for integer N, extend these relations to arbitrary N, and find objects (not necessarily $N \times N$ matrices) satisfying them.

In trying to address the same question for models where $\sigma_1 \neq 0$, it is natural to set up the problem in algebraic terms again. The objects E, P, I can be understood as providing a particular representation of the Birman-Wenzl algebra [19]. (The definition of this algebra can also be found in [15].)

Although it seems to be problematic to extend the definition of the S matrix to arbitrary values of N, it is easy nevertheless to extend it to negative integer values of N. Indeed, the Birman-Wenzl algebras arise from the representation theory of $O(N)$, and most of the properties of these algebras generalize to the superalgebras $OSP(m/2n)$. Instead of the vector representation of $O(N)$, take the vector representation of the orthosymplectic algebra, of dimensions $(m, 2n)$. For $m \neq 2n$, the tensor product with itself gives rise to three representations. Taking I as the identity, E as $(m - 2n)$ times the projector on the identity representation, and P as the graded permutation operator (the extension to the case $m = 2n$ is easy), it can be checked indeed that the defining relations of the Birman- Wenzl algebra are obeyed with $N = m - 2n$.

Leaving aside the unitary difficulty, the usual formal procedure selects once again the factors $\sigma_2^{\pm}$ as minimal prefactors, with the continued values $N = m - 2n$. The question is then to establish the relations with field theory, if any.

Taking the OSP $\check{S}$ matrix, and the S matrix that follows from it ($\check{S} = PS$ where P is the graded permutation operator), $S = \sigma_1 E + \sigma_2 I + \sigma_3 P$, it is natural to ask about the physical meaning of these amplitudes. This reveals some surprises. Crossing and unitarity are well implemented in the cases when the particles are bosons or fermions. Mixing the two kinds does not seem, a priori, to give rise to any difficulty. It will turn out however that in the graded case, the S matrix is, as a matrix, **not unitary.** It is thus difficult to interpret our S matrices in terms of a "physical" scattering. The most useful way to think of the S matrices will probably be as an object describing the monodromy of wave functions, like in imaginary Toda theories [13, 20]. Crossing follows then from $\check{S}(i\pi-\theta)=\sigma_1(\theta)I+\sigma_2(\theta)P+\sigma_3(\theta)E$, with an obvious graphical interpretation, and charge conjugation being defined through the defining form of the OSP algebra.

3. Scattering theory with σ_2^+ and the sphere sigma model

3.1 $OSP(1/2)$ with σ_2^+ and the $a_2^{(2)}$ Toda theory

Let us now consider the "scattering" theory that is the continuation of the sphere sigma model to $N=-1$: we take the $OSP(1/2)$ realization, and as a prefactor σ_2^+.

It then turns out that the S matrix is identical to the one of the $a_2^{(2)}$ Toda theory for a particular value of the coupling constant! This will allow us to explicitly perform the TBA, and identify the scattering theory indeed. While we were carrying out these calculations, we found out two papers where the idea has been carried out to some extent already: one by Martins [21], and one by Sakai and Tsuboi [22]. Our approach has little overlap with these papers, and stems from our earlier work on the $a_2^{(1)}$ theory instead.

Let us first introduce the scattering matrices for the two theories we will identify with each other:

On the one hand, the solution of the graded Yang Baxter equation relevant here is the well known $OSP(1/2)$ one, given by:

$$R_{OSP(1/2)}=\frac{1}{1-3\frac{\theta}{2i\pi}}\left[P+\frac{3\theta}{2i\pi}I+\frac{\theta}{i\pi-\theta}E\right], \tag{5}$$

where we have chosen the normalization factor for later purposes, I is the identity. Denote the basis vectors in the fundamental representation of $OSP(1/2)$ as b, f_1, f_2. The operators E and P are given by the following matrices:

$$E=\begin{pmatrix}1 & -1 & 1\\ 1 & -1 & 1\\ -1 & 1 & -1\end{pmatrix},\qquad P=\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & -1\\ 0 & -1 & 0\end{pmatrix}, \tag{6}$$

in the subspace spanned by $(b,b),(f_1,f_2),(f_2,f_1)$ in that order, $E=0$ otherwise. The operators E,P satisfy the defining relations of the Birman-Wenzl algebra with $N=-1$.

On the other hand, we will consider the $a_2^{(2)}$ Toda theory. It has the action:

$$S = \frac{1}{8\pi} \int dx dy \left[(\partial_x \Phi)^2 + (\partial_y \Phi)^2 + \Lambda (2e^{-\frac{i}{\sqrt{2}}\beta\Phi} + e^{i\sqrt{2}\beta\Phi}) \right] . \quad (7)$$

The "effective" dimension of the perturbation is $d = \beta^2$ and the region we shall primarily be interested corresponds to $\beta^2 \geq 1$. We will parametrize $\beta^2 = 2\frac{t-1}{t}$ so $h_1 = \frac{\beta^2}{4} = \frac{t-1}{2t}$, and we will restrict ourselves to the region $\Lambda > 0$ in the following. In the $t \in [2, \infty]$ domain, the scattering matrix has been first conjectured by Smirnov [23]. The $\check{S}$ matrix is proportional to the $\check{R}$ matrix of the Izergin Korepin model [24].

The correspondence between the scattering theories of $OSP(1/2)$ and $a_2^{(2)}$ results from the observation that the $a_2^{(2)}$ $\check{S}$ matrix can be identified in the limit $t \to \infty$ with the $OSP(1/2)$ "sphere sigma-model" $\check{S}$ matrix corresponding to the expression given in (5) [15].

This coincidence has a simple algebraic origin. Indeed recall [25, 26], that the $a_2^{(2)}$ Toda theory has symmetry $U_q(a_2^{(2)})$, $q = e^{i\pi/\beta^2}$. The Dynkin diagram for the algebra $a_2^{(2)}$ turns out to be almost identical to the one for the algebra $osp(1|2)^{(1)}$, although in the latter case, one of the roots is fermionic, and therefore the basic relations involve an anticommutator instead of a commutator. It can be shown that for a particular value of q, namely $q = i$, and an additional restriction which is satisfied in the case at hand, the q-deformation of one algebra gives rise to the other, so that there is a mapping between $U_q(a_2^{(2)})$ and $U(osp(1|2)^{(1)})$, for $q = i$. This should not come as a surprise, and has algebraic roots going back as far as [27]. For recent related works, see [28, 29].

Throughout this paper, we will use the thermodynamic Bethe ansatz to calculate physical properties of our theory. It is a priori unclear whether the method - which involves maximizing a free energy - makes much sense in a theory whose Hamiltonian is not Hermitian, but the results we obtain seem perfectly meaningful, like in other similar examples. String solutions for the $a_2^{(2)}$ model were not known before, but they can easily be obtained using our recent results on the $a_2^{(1)}$ case. Setting $\gamma = \frac{\pi}{t-1}$, the $a_2^{(2)}$ Bethe equations have the form:

$$\prod_{\alpha} \frac{\sinh \frac{1}{2}(y_i - u_\alpha - i\gamma)}{\sinh \frac{1}{2}(y_i - u_\alpha + i\gamma)} = \prod_{j} \frac{\sinh \frac{1}{2}(y_i - y_j - 2i\gamma)\sinh \frac{1}{2}(y_i - y_j + i\gamma)}{\sinh \frac{1}{2}(y_i - y_j + 2i\gamma)\sinh \frac{1}{2}(y_i - y_j - i\gamma)},$$

where the y_i are Bethe roots, and the u_α are spectral parameter heterogeneities (corresponding to the rapidities of particles already present in the system). The solutions of these equations in the thermodynamic limit are as follows. The y's can be $1, 2, \ldots, t-1$ strings, or antistrings. In addition, it is possible to have a t string centered on an antistring, or to have a complex of the form

$y = y_r \pm \frac{i\gamma}{2} + i\pi$. The solutions for the more general case of twisted boundary conditions are given in [15].

The Bethe ansatz equations for $a_2^{(2)}$ in the $t \to \infty$ limit, with $y = \gamma\lambda$, $u = \gamma\mu$, $\gamma \to 0$ match the Bethe Ansatz equations for the $OSP(1/2)$ model given in [30]:

$$\prod \frac{\lambda_i - \mu_\alpha - i}{\lambda_i - \mu_\alpha + i} = \prod \frac{\lambda_i - \lambda_j - 2i}{\lambda_i - \lambda' + 2i} \prod \frac{\lambda_i - \lambda_j + i}{\lambda_i - \lambda_j - i} . \tag{8}$$

Twisting and truncating the $a_2^{(2)}$ models one can obtain the RSOS models $M_{t-1,t/2}$ for t even and $M_{t,(t-1)/2}$ for t odd.

3.2 The $OSP(1/2)$ limit of $a_2^{(2)}$ and the sphere sigma model

It is also possible to establish a correspondence between the actions of the $OSP(1/2)$ limit of $a_2^{(2)}$ and the sphere sigma model. After identifying the twisted Toda theory in the limit $t \to \infty$ with symplectic fermions, the corresponding action can be rescaled and can be brought into the form:

$$S = \int \frac{d^2x}{2\pi} \left[\partial_\mu\eta_1\partial_\mu\eta_2 + \Lambda\eta_1\eta_2\partial_\mu\eta_1\partial_\mu\eta_2\right] , \tag{9}$$

where the coupling Λ is positive.

On the other hand, the action of the sphere sigma model can generally be written using bosonic and fermionic coordinates of the supersphere $S^{n-1,2n}$. In the $S^{0,2}$ case, the action can be shown to be [15]:

$$S = -\frac{1}{|g|} \int d^2x \left[\partial_\mu\eta_1\partial_\mu\eta_2 - \frac{1}{2}\eta_1\eta_2\partial_\mu\eta_1\partial_\mu\eta_2\right] . \tag{10}$$

Note that a rescaling combined with a relabeling can always bring this action into the form (9) with $\Lambda \propto |g|$, identifying the two actions.

3.3 Supersphere sigma models with $OSP(1/2n)$ symmetry and $a_{2n}^{(2)}$

The relation we uncovered between $a_2^{(2)}$ and $OSP(1/2)$ extends immediately to the case of $a_{2n}^{(2)}$ and $OSP(1/2n)$: one can establish, for general values of n, the relation between the quantum affine algebras, the Bethe ansatz equations, the scattering matrices etc. We thus propose that the S matrix with $OSP(1/2n)$ symmetry, represented in (1),(3),(4) with $N = 1 - 2n$, and the prefactor σ_2^+, provides an analytic continuation of the $O(N)/O(N-1)$ "sphere" sigma model to this value of N where the analytic continuation of the sigma model should be interpreted as the coset $OSP(1/2n)/OSP(0/2n)$. The effective central charge of the UV limit is $c_{\text{eff}} = n$, while its true central charge will be $c = -2n$. For the ordinary sigma models, the UV central charge is $N-1$, so the UV value in the analytic continuation just matches.

It is also possible to extend the analysis of the $a_2^{(2)}$ TBA to arbitrary value of n, so we also know the TBA for this scattering theory, which is simply given by a Z_2 folding of the $a_{2n}^{(1)}$ TBA. The corresponding TBA diagram can be found in [15].

Our results have an immediate application to the study of quantum spin chains. Indeed, the Bethe equations which appear in the solution of the $OSP(1/2n)$ sigma models are similar to the ones appearing in the solution of the integrable $OSP(1/2n)$ chains studied in particular by Martins and Nienhuis [31]. More detailed calculations show that these chains are critical, and that they coincide at large distance with the weakly coupled supersphere sigma models, that is, a system of $2n$ free symplectic fermions. This is in disagreement with the conjecture in [21, 31] that this continuum limit should be a WZW model on the supergroup: although the central charge agrees with both proposals, detailed calculations of the thermodynamics or finite size spectra show that the WZW proposal is not correct, and confirm the sigma model proposal instead. A similar conclusion holds for $OSP(m/2n)$ when $m-2n<2$. That the spin chain flows to the weakly coupled sigma model is certainly related with the change of sign of the beta function when $m-2n$ crosses the value 2, but we lack a detailed understanding of the mechanisms involved.

4. Scattering theory with σ_2^- and super Gross-Neveu models

4.1 Gross-Neveu and WZW models

If we consider a scattering matrix defined again by (1),(3), (4) but now with the prefactor σ_2^- instead, it is natural to expect that it describes $OSP(m/2n)$ Gross-Neveu models, the analytic continuation of the $O(N)$ GN models to $O(m-2n)$. The $OSP(m/2n)$ Gross-Neveu models read:

$$S = \int \frac{d^2x}{2\pi}\left[\sum_{i=1}^{m} \psi_L^i \partial \psi_L^i + \psi_R^i \bar{\partial} \psi_R^i + \sum_{j=1}^{n} \beta_L^j \partial \gamma_L^j + \beta_R^j \bar{\partial} \gamma_R^j \right.$$
$$\left. + g\left(\psi_L^i \psi_R^i + \beta_L^j \gamma_R^j - \gamma_L^j \beta_R^j\right)^2\right], \tag{11}$$

where the ψ are Majorana fermions of conformal weight $1/2$, and the $\beta\gamma$ are bosonic ghosts of weight $1/2$ as well. This theory has central charge $c = \frac{m}{2} - n$, effective central charge $c_{\text{eff}} = \frac{m}{2} + 2n$. Perturbative calculations of the beta function [22, 32] suggest that this model behaves like the continuation of the $O(N)$ Gross-Neveu model to the value $N = m - 2n$: The beta function for the model (11) is of the form $\beta_g \propto (m - 2n - 2)g^2$, the same as the one for the $O(m-2n)$ GN model. For $m-2n > 2$, it is thus positive, so a positive coupling g is marginally relevant - this is the usual massive GN model - while a negative one is marginally irrelevant. If instead we consider the case $m - 2n < 2$, these results are switched: it is a negative coupling that is marginally relevant, and makes the theory massive in the IR [1]. The case $m = 1$ should be described by the foregoing scattering theory.

Note that the GN model is equivalent to the appropriate WZW model with a current current perturbation. Indeed, the system of m Majorana fermions and n symplectic bosons constitutes in fact a certain representation of the $OSP(m/2n)$ current algebra where the level depends on the choice of normalization.

4.2 The $OSP(0/2)$ case

The simplest case is the GN model for $N = -2$, corresponding formally to $OSP(0/2)$, i.e. a $\beta\gamma$ system. The scattering matrix for this system turns out to be $\check{S} = i \tanh\left(\frac{\theta}{2} - \frac{i\pi}{4}\right) \check{S}_{SG}(\beta^2_{SG} = 8\pi)$ where S_{SG} is the soliton S matrix of the sine-Gordon model. At coupling $\beta^2_{SG} = 8\pi$, it coincides with the S matrix of the $SU(2)$ invariant Thirring model, or the level 1 WZW model with a current current perturbation. The scattering matrix is thus the same as the one for the $k = 1$ $SU(2)$ WZW model up to a CDD factor. This CDD factor does not introduce any additional physical pole, but affects the TBA in an essential way. In fact, the study of the TBA for the anisotropic deformation of the model (the same that was used in the sigma model case) reveals a surprise. For a particular value of the fugacity (namely $e^{i\pm\pi}$), and taking the limit $t \to \infty$, the central charge is $c = -\infty$! This fact can be explained by the existence of zero modes which render the $\beta\gamma$ system unstable. It can be shown that adding a mass term (which is actually a current-current perturbation) in the $OSP(0/2)$ GN model stabilizes the theory [15].

4.3 The $OSP(1/2n)$ case

We now consider the $OSP(1/2n)$ case. The TBA turns out to have a simple description in terms of $a^{(2)}_{2n}$ again. Consider therefore, not the $SU(2n+1)$ GN model, but a related scattering theory with only two multiplets of particles, corresponding respectively to the defining representation and its conjugate. Considering more generally the case of $SU(P)$ models, the relation between

[1] In [11], the four fermion coupling is defined through combinations $\bar{\psi}_-\psi_+ + \psi_-\bar{\psi}_+ = 2i(\psi^1_L\psi^1_R + \psi^2_L\psi^2_R)$, so what is called g there is the opposite of our convention.

the $SU(P)$ GN scattering theory and this new theory is similar to the relation between the $O(P)$ GN model and the $O(P)/O(P-1)$ sigma model [33]. We will thus call this scattering theory "sigma model like", but we are not aware of any physical interpretation for it. The TBA can be found in [15] and is quite similar to the one of the $\mathcal{N}=2$ supersymmetric $SU(P)$ Toda theory [34] (the generalization of the supersymmetric sine-Gordon model for $SU(2)$): the difference affects only which nodes correspond to massive particles, and which ones to pseudo particles. As a result, the central charge is easily determined, $c = 2P-1$. Getting back to the particular case $P = 2n+1$, we can then fold this system to obtain (see [15] for the proof) the TBA for the $OSP(1/2n)$ Gross-Neveu model, whose effective central charge reads therefore $c_{\text{eff}} = \frac{1}{2}(2(2n+1)-1) = 2n+\frac{1}{2}$.

5. Finite field calculations

Further evidence for our S matrices can be obtained from finite field calculations. The idea, which has been worked out in great details in other cases [35], is to compare S matrix and perturbative calculations for the ground state energy of the theory in the presence of an external field.

It turns out [15] that the ground state energy of the $OSP(m/2n)$ Gross-Neveu model and the $OSP(m/2n)$ sigma model can be obtained easily from known expressions for the $O(N)$ sphere sigma model and the $O(N)$ Gross-Neveu model, respectively (the roles of the two models are interchanged here).

From these results we can compute the ratio of the first two coefficients of the beta function as $\frac{\beta_2}{\beta_1^2} = \frac{1}{N-2}$. The ratios we found turn out to be the analytic continuations to $N \to m-2n$ in the two respective cases, as desired.

6. Conclusions and speculations

To conclude, although more verifications ought to be carried out to complete our identifications, we believe we have determined the scattering matrices for the massive regimes of the $OSP(m/2n)$ GN and the $OSP(m/2n)/OSP(m-1/2n)$ sigma models in the simple case $m=1$, based on algebraic considerations as well as thermodynamic Bethe ansatz calculations.

It is tempting to expect that at least some of our results generalize to other cases $OSP(m/2n)$ for $m>1$ and $m-2n<2$. In all these cases, we expect that the S matrix of the sphere sigma model will be obtained from the conjecture at the beginning of this paper, with $N = m-2n$, for $N<2$. The S matrix of the GN model is probably more complicated. Recall that in the case $N \geq 2$, it is given by the general conjecture only for $N>4$. When $N \leq 2$, we think it is probably given by the conjecture only for $N<0$.

Besides completing the identifications we have sketched here, the most pressing questions that come to mind are: what are the S matrices of the Gross-Neveu models for non–generic values of N, what are the S matrices for the multiflavour GN models, what are the S matrices for the orthosymplectic Principal Chiral Models? We hope to report some answers to these questions soon.

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