

Math/Phys Seminar

Who: Yu Li (University of Notre Dame)

Where: Tuesday Oct 28, 1:30-2:30 pm, SC G060

Title:

Cluster structures, integrable systems and symplectic groupoids.

Abstract:

We introduce two operations that can be applied to a compatible cluster structure Φ on a Poisson variety (Y, π) . (1) If there exists a point $y \in Y$ where the Poisson bivector π vanishes, then taking the lowest degree terms of Φ gives rise to a set Φ^{low} of pairwise Poisson commutative functions on the tangent space $T_y Y$ equipped with the Poisson bivector π_0 which is the linearization of π . We present a sufficient condition, in terms of the degree of the log-volume form of Φ , under which Φ^{low} is an integrable system on $(T_y Y, \pi_0)$. When (Y, π) is the Poisson dual group of the standard Poisson Lie group GL_n and Φ is the generalized cluster structure of Gekhtman-Shapiro-Vainshtein, our operation produces an integrable system on $\mathfrak{gl}_n^*$ which is different than the celebrated Gelfand-Zeitlin integrable system. (2) If the Poisson variety (Y, π) integrates into a symplectic groupoid $(\mathcal{G}, \Omega) \implies (Y, \pi)$, then $s^* \Phi \cup t^* \Phi$ is a set of log-canonical functions on $(\mathcal{G}, \Omega)$, which, in many examples of representation theoretical interest, can be completed to a compatible cluster structure Ψ . We explain how the mutable variables, frozen variables and cluster mutations of Ψ are related to those of Φ . When (Y, π) is the standard Poisson Lie group GL_n , our operation produces a compatible cluster structure on the Heisenberg double of GL_n . Time permitting, we explain how to relate these two operations by taking the lowest degree terms of Ψ along the identity section of $\mathcal{G}$.