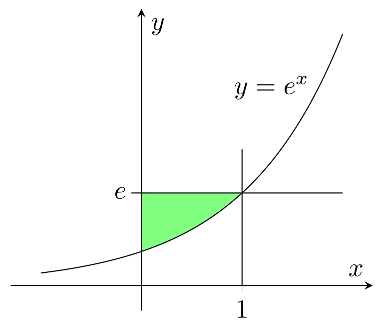
Instructions. Show all work, with clear logical steps. No work or hard-to-follow work will lose points.

Problem 1. (3 points) Switch the order of integration of the following integral.

$$\int_0^1 \int_{e^x}^e f(x,y)\,dy\,dx$$

Solution. Of course we start by drawing a picture of the domain of integration.



So to switch the order of integration, we want to find the limits of integration for x first. Here, the smallest x can be is at the y-axis, and the largest x can be is at the exponential curve. Of course, the y-axis is just $x = 0$, and solving $y = e^x$ for x, we get $x = \ln y$. Finding the limits of integration for y is easier. The smallest y can be is 1 and the largest y can be is e. So switching the bounds of the original integral, we get

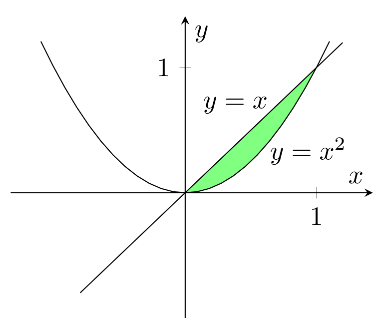
$$\int_1^e \int_0^{\ln y} f(x,y)\,dx\,dy.$$

□

Problem 2. (3 points) Switch the order of integration of the following integral.

$$\int_0^1 \int_{x^2}^x f(x,y)\,dydx$$

Solution. Start by drawing a picture of the domain of integration.



We want our to curves to be functions of y in order to switch the order of integration. The line is already taken care of. For the parabola, solving for x we get $x = \sqrt{y}$ (just the positive one since we are in the first quadrant).

The smallest x can be is on the line and the largest x can be is on the parabola. So x ranges from y to $\sqrt{y}$. And y ranges from 0 to where the line and the parabola intersect, which is at $x = 1$. So when we switch the order of integration, we get

$$\int_0^1 \int_y^{\sqrt{y}} f(x,y)\,dx\,dy.$$

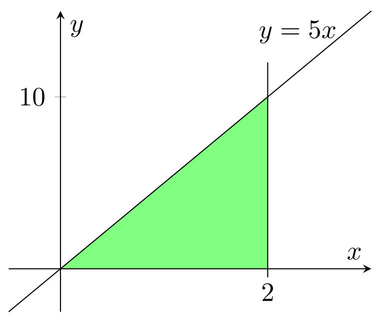
□

Problem 3. (4 points) Evaluate

$$\iint_D \frac{1}{x^2+7}\,dA,$$

where D is the region bounded by $y = 5x$, the x-axis and $x = 2$.

Solution. We start by drawing a picture of D.



Notice that y ranges from the x-axis and the line $y = 5x$ and x ranges from 0 to 2. Putting this in the double integral,

$$\begin{aligned}
\iint_D \frac{1}{x^2+7}\,dA &= \int_0^2 \int_0^{5x} \frac{1}{x^2+7}\,dy\,dx \\
&= \int_0^2 \frac{5x}{x^2+7}\,dx && u = x^2+7, \quad du = 2x\,dx \\
&= \frac{5}{2}\int_7^1 1\,\frac{du}{u} \\
&= \frac{5}{2}\ln u\Big|_7^{11} \\
&= \frac{5}{2}(\ln 11 - \ln 7) \\
&= \frac{5}{2}\ln\frac{11}{7}.
\end{aligned}$$

□