Instructions. Show all work, with clear logical steps. No work or hard-to-follow work will lose points.

Problem 1. (4 points) Solve for y as a function of t when

$$y' = -\frac{6t}{y}.$$

Solution. We start by getting all the y's on the left and the t's on the right.

$$\begin{aligned}\frac{dy}{dt} &= -\frac{6t}{y}\\ y\,dy &= -6t\,dt\\ \int y\,dy &= \int -6t\,dt\\ \frac{1}{2}y^2 &= -3t^2 + C\\ y^2 &= -6t^2 + C\\ &\boxed{y = \pm\sqrt{-6t^2 + C}}\end{aligned}$$

□

Problem 2. (3 points) Compute

$$\int \frac{\ln x}{x^2}\,dx$$

Solution. It's good to try u-substitution first, but $u = \ln x$ and $u = 1/x^2$ don't work. So this is an integration by parts problem. Following 'LATE', we pick

$$u = \ln x \qquad dv = \frac{1}{x^2}\,dx$$

$$du = \frac{1}{x}\,dx \qquad v = -\frac{1}{x}$$

Then $\int u\,dv = uv - \int v\,du$, so

$$\begin{aligned}\int \frac{\ln x}{x^2}\,dx &= -\frac{1}{x}\ln x - \int -\frac{1}{x^2}\,dx\\ &= -\frac{1}{x}\ln x + \int \frac{1}{x^2}\,dx\\ &= \boxed{-\frac{1}{x}\ln x - \frac{1}{x} + C}\end{aligned}$$

□

Problem 3. (3 points) Compute

$$\int \frac{(\ln x)^2}{x}\,dx$$

Solution. We try u-substitution first. The natural choice for u is $u = \ln x$. Then $du = \frac{1}{x}\,dx$. So

$$\begin{aligned}\int \frac{(\ln x)^2}{x}\,dx &= \int u^2\,du \\ &= \frac{1}{3}u^3 + C \\ &= \boxed{\frac{1}{3}(\ln x)^3 + C}\end{aligned}$$

$\square$