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STRUCTURAL STABILITY IN SOME FAMILIES OF ENTIRE FUNCTIONS

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UDC 517.53+517.98

Let f be an entire function. Let R(f) be the maximum open set on which the family of iterations $\{f^n\}$ is normal in the sense of Montel. It is well known that the Julia set $J(f) = \mathbf{C}\setminus R(f)$ is nonempty and perfect [1]. Sullivan [2] gave a complete description of the asymptotic behavior of the iterations of a rational function on a set of normality. An analogous description was given by the authors for a class S of entire transcendental functions [3]. We say that an entire function belongs to the class S_q if there exists a finite set of points $\{a_1,\ldots,a_q\}$ such that $f\colon \mathbf{C}\setminus f^{-1}(\{a_1,\ldots,a_q\})\to\mathbf{C}\setminus\{a_1,\ldots,a_q\}$ is a nonramified covering. A minimum set of points with this property is spoken of as a set of base points of the function f. We put $S=\bigcup_{q=1}^{\infty} S_q$. This class is closed with respect to superpositions. Examples: $\exp\in S_1$, $\sin\in S_2$. If f and g are polynomials of degrees m and n, respectively, it follows that $f\in S_{m-1}$,

$$\int f(\zeta)\exp g(\zeta)\,d\zeta\in S_{m+n}.$$

We say that the entire functions f and g are equivalent if homeomorphisms $\varphi,\ \psi\colon \mathbf{C}\to\mathbf{C}$ exist such that $\psi\circ f=g\circ\varphi$. Let $g\in S_q\setminus S_{q-1}$ for some $q\geqslant 1$ and let M = M(g) be an equivalence class containing the function g. On the set M we can introduce the structure of a (q + 2)-dimensional complex analytic manifold so that the mapping

$$\mathbf{C}\times M\to\mathbf{C},\quad (z,\ f)\mapsto f(z)$$

is analytic in both variables. As local coordinates we can choose the base points of the function f and its values at two points. The topology in M coincides locally with the topology of uniform convergence on compacta in $\mathbf{C}$. We note that polynomials of general position of degree n form a manifold M of dimension n − 1.

Let $f\in M$. We consider an equation defining the periodic points α of the function f:

$$f^p(\alpha)=\alpha. \tag{1}$$

The solution of this equation is a many-valued function α on the manifold M.

THEOREM 1. The function α has on M only algebraic singularities.

The function $f\in M$ is said to be J-stable if for an arbitrary function $f_1\in M$, close to f, we can find a homeomorphism $h\colon J(f)\to J(f_1)$ such that $h\circ f=f_1\circ h$ on J(f).

If α is a solution of Eq. (1), the set $\{\alpha, f\alpha,\ldots,f^{p-1}\alpha\}$ is called a cycle and p its period. If p is the smallest of the periods, the number $(f^p)'(\alpha)$ is called the multiplicator of the cycle.

Let N be a set of functions $f\in M$ having a cycle whose multiplicator is a root of 1.

THEOREM 2. The set $\Sigma = M\setminus\overline{N}$ is everywhere dense in M. The functions $f\in\Sigma$ are J-stable.

Physicotechnical Institute of Low Temperatures, Academy of Sciences of the Ukrainian SSR, Kiev. Translated from Funktsional'nyi Analiz i Ego Prilozheniya, Vol. 19, No. 4, pp. 86-87, October-December, 1985. Original article submitted June 25, 1984.

0016-2663/85/1904-0323$09.50

This theorem was proved by the method used in [4-6] with the aid of Theorem 1.

A function $f \in M$ is said to be structurally stable if for an arbitrary function $f_1 \in M$, close to f, we can find a homeomorphism $h: \mathbf{C} \to \mathbf{C}$ such that $h \circ f = f_1 \circ h$.

Let $f \in M$; and let $a_1, \ldots, a_q$ be base points of the function f. We denote by Λ the set of $f \in M$ such that $f^k(a_j) = f^l(a_i)$ for certain distinct pairs (l, i) and (k, j).

THEOREM 3. The set $\Sigma \setminus \Lambda$ is open and everywhere dense in M. The functions $f \in \Sigma \setminus \Lambda$ are structurally stable. The joining homeomorphisms are quasiconformal in **C**.

The proof of Theorem 3 employs the method used in [5], certain results from [3], and Theorem 2.

The simplest family M consists of functions of the form $a \exp(bz) + c$. Iterations of these entire functions have been studied for some time. We mention only some of the most recent studies: [3, 7, 8].

Entire functions, conjugate through the linear transformation $z \to az + b$, have identical dynamic properties. It is therefore sufficient to restrict ourselves to the single-parameter family $f_c(z) = \exp z + c$. It follows from Theorem 3 that the function f_c is structurally stable when c belongs to an open everywhere dense set. It is not difficult to show that for an arbitrary positive integer p, we can find an open set D_p such that for all $c \in D_p$ the function f_c has a unique cycle of order p with multiplicator $\lambda(c)$, $|\lambda(c)| < 1$.

THEOREM 4. Each component $\varkappa$ of the set D_p is simply connected and unbounded. The mapping

$$\lambda : \varkappa \to \{z : 0 < |z| < 1\}$$

is a universal covering.

This theorem is an analog of a theorem of Douady and Hubbard [9], who studied the family $z^2 + c$.

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