Lesson 14: Derivatives of Logarithmic and Exponentials

10:27 AM

Watch the following video until 5 mins: [Derivatives of Logarithmic and Exponential Functions]

Recap:

$y = f(x)$	$y' = dy/dx$
$y = \ln(x)$	$y' = 1/x$
$y = \ln(\lvert x \rvert)$	$y' = 1/x$
$y = \log_a(x) = \frac{\ln(x)}{\ln(a)}$	$y' = \frac{1}{\ln(a)} \cdot (\ln(x))' = \frac{1}{\ln(a)} \cdot \frac{1}{x}$
$y = e^x$	$y' = e^x$
$y = a^x$	$y' = a^x \ln(a)$

Example 1: Find y' of

(a) $y = 3^x$

Rule: $y' = a^x \ln(a)$

$y' = 3^x \ln(3)$

(b) $y = \frac{7^x}{7^x + 1}$

$u = 7^x \qquad v = 7^x + 1$

$u' = 7^x \ln(7) \qquad v' = 7^x \ln(7) + 0$

Quotient Rule: $y' = \frac{u'v - uv'}{v^2}$

$$y' = \frac{7^x \ln(7)[7^x + 1] - 7^x[7^x \ln(7)]}{(7^x + 1)^2}$$

$$= \frac{\cancel{7^x \cdot 7^x \ln(7)} + 7^x \ln(7) - \cancel{7^x \cdot 7^x \ln(7)}}{(7^x + 1)^2} = \frac{7^x \ln(7)}{(7^x + 1)^2}$$

$$= \frac{[\text{illegible}]}{(7^x+1)^2} = \frac{[\text{illegible}]}{(7^x+1)^2}$$

Example 2: Find y'.

① $y = \ln(x^5+2)^{2\pi}$

Rule: $y = \ln(x^m) = m\ln(x)$

Rewrite $y = 2\pi \ln(x^5+2)$

$$y' = 2\pi \cdot \frac{1}{x^5+2}(x^5+2)'$$

$$y = \frac{2\pi\,(5x^4)}{x^5+2}$$

derivative

② $y = \ln(\ln(2x)) \neq \ln^2(2x)$

derivative

$$y' = \frac{1}{\ln(2x)} \cdot (\ln(2x))'$$

$$= \frac{1}{\ln(2x)} \cdot \frac{1}{2x} \cdot (2x)'$$

$$= \frac{1}{\ln(2x)} \cdot \frac{1}{2x} \cdot 2 = \frac{1}{x\ln(2x)}$$

③ $y = \ln\left(\frac{2x-1}{2x+1}\right)$

Rule $y = \ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y)$

Rewrite $y = \ln(2x-1) - \ln(2x+1)$

$$= \frac{1}{2x-1}(2x-1)' - \frac{1}{2x+1}(2x+1)'$$

$$= \frac{2}{2x-1} - \frac{2}{2x+1}$$

Example 3: Find y' for

$$y = \frac{(x^3-1)^4\sqrt{3x-1}}{x^2+4}$$

$$\frac{\qquad\qquad}{x^2+4}$$

Take ln of both sides.

$$\ln(y) = \ln\left(\frac{(x^3-1)^4(3x-1)^{1/2}}{x^2+4}\right)$$

$$= \ln\left((x^3-1)^4(3x-1)^{1/2}\right) - \ln(x^2+4)$$

$$= \ln\left((x^3-1)^4\right) + \ln\left((3x-1)^{1/2}\right) - \ln(x^2+4)$$

$$\ln(y) = 4\ln(x^3-1) + \frac{1}{2}\ln(3x-1) - \ln(x^2+4)$$

$$\frac{d}{dx}(\ln(y)) = \frac{d}{dx}\left[4\ln(x^3-1) + \frac{1}{2}\ln(3x-1) - \ln(x^2+4)\right]$$

$$\frac{1}{y}\boxed{\frac{dy}{dx}} = \left(\frac{4}{x^3-1}\cdot(3x^2) + \frac{1}{2}\cdot\frac{1}{3x-1}\cdot(3) - \frac{1}{x^2+4}\cdot 2x\right)\cancel{\frac{dx}{dx}}^{\,1}$$

Solve for this

$$\frac{dy}{dx} = y\left(\frac{12x^2}{x^3-1} + \frac{3}{2}\cdot\frac{1}{3x-1} - \frac{2x}{x^2+4}\right)$$

$$= \frac{(x^3-1)^4\sqrt{3x-1}}{x^2+4}\left(\frac{12x^2}{x^3-1} + \frac{3}{2}\cdot\frac{1}{3x-1} - \frac{2x}{x^2+4}\right)$$