Lesson 16: Derivatives of Inverse Trig Functions

Monday, October 6, 2025 10:23 AM

Oct 27–31: Virtual Class

Derivatives of Inverse Trig Functions

For this lesson, we only want to find the derivatives of

$$\sin^{-1}(x) = \arcsin(x)$$
$$\cos^{-1}(x) = \arccos(x)$$
$$\tan^{-1}(x) = \arctan(x)$$

Never $\sin^{-1}(x) \neq \frac{1}{\sin(x)}$ Bad math

Determine the derivative of $\sin^{-1}(x)$.

Let $\sin^{-1}(x) = \theta = y$

$$x = \sin\theta$$

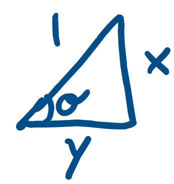
Idea is to get $\frac{dy}{dx}$

$$\frac{d}{dx}(x) = \frac{d}{dx}(\sin\theta)$$

$$1 = \cos\theta \frac{d\theta}{dx}$$

$$\frac{1}{\cos\theta} = \frac{d\theta}{dx} = \frac{dy}{dx}$$

Remember $\frac{x}{1} = \sin\theta$



Find y.

$$x^2 + y^2 = 1^2$$
$$y^2 = 1 - x^2$$
$$y = \sqrt{1-x^2}$$

$$\frac{dy}{dx} = \frac{1}{\cos\theta} = \frac{1}{\sqrt{1-x^2}}$$

To get x terms on the left I need the triangles.

Formulas:

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}} \text{ for } -1 < x < 1$$

$$\frac{d}{dx}(\cos^{-1}(x)) = -\frac{1}{\sqrt{1-x^2}} \text{ for } -1 < x < 1$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2} \text{ for } -\infty < x < \infty$$

Find the derivative of the following

Ex 1: Find the derivative of the following

ⓐ $f(x) = \sin^{-1}(-x)$

Chain Rule

$$f'(x) = \frac{1}{\sqrt{1-(-x)^2}} \cdot \frac{d}{dx}(-x)$$

$$= \frac{1}{\sqrt{1-x^2}} \cdot (-1) = \frac{-1}{\sqrt{1-x^2}}$$

ⓑ $f(x) = \tan^{-1}(7x^5+1)$

Chain Rule

$$f'(x) = \frac{1}{1+(7x^5+1)^2} \frac{d}{dx}(7x^5+1)$$

$$= \frac{35x^4}{1+(7x^5+1)^2}$$

ⓒ $f(x) = (\cos^{-1}(x))^{21}$

Chain Rule

$$f'(x) = 21(\cos^{-1}(x))^{20} \cdot \frac{d}{dx}(\cos^{-1}(x))$$

$$= 21(\cos^{-1}(x))^{20}\left(\frac{-1}{\sqrt{1-x^2}}\right)$$

ⓓ $f(x) = \sin^{-1}(e^{8\sin(x)})$

Chain Rule w/in Chain Rule

$$f'(x) = \frac{1}{\sqrt{1-(e^{8\sin(x)})^2}} \frac{d}{dx}(e^{8\sin(x)})$$

$$= \frac{1}{\sqrt{1- e^{16\sin(x)}}} \cdot e^{8\sin(x)} \frac{d}{dx}(8\sin(x))$$

$$= \frac{1}{\sqrt{1-e^{16\sin(x)}}} \cdot e^{8\sin(x)}\, 8\cos(x)$$