MA 165 LESSONS 17-18: RELATED RATES

Example 1: If x and y are both functions of t and $x + y^3 = 2$.

(a) Find $\frac{dy}{dt}$ when $\frac{dx}{dt} = -2$ and $y = 1$.

(b) Find $\frac{dx}{dt}$ when $\frac{dy}{dt} = 3$ and $x = 1$.

Recipe for Solving a Related Rates Problem

Step 1: Draw a good picture. Label all constant values and give variable names to any changing quantities.

Step 2: Determine what information you **KNOW** and what you **WANT** to find.

Step 3: Find an equation relating the relevant variables. This usually involves a formula from geometry, similar triangles, the Pythagorean Theorem, or a formula from trigonometry. **Use your picture!**

Step 4: Use implicit differentiation to differentiate the equation with respect to time *t.*

Step 5: Substitute in what you **KNOW** from **Step 2** and any information that your equation in **Step 3** can give you and solve for the quantity you **WANT**. Do **NOT** substitute before this step!

Some Useful Formulas

Right Triangle *Pythagorean Theorem:* $a^2 + b^2 = c^2$	Triangle $A = \frac{1}{2}bh$ $P = a + b + c$	Trapezoid $A = \frac{1}{2}(a + b)h$
Rectangular Box $V = lwh$ $S = 2(hl + lw + hw)$	Rectangle $A = lw$ $P = 2l + 2w$	Circle $A = \pi r^2$ $C = 2\pi r$
Right Circular Cylinder $V = \pi r^2 h$ $SA = 2\pi rh$	Sphere $V = \frac{4}{3}\pi r^3$ $S = 4\pi r^2$	Cone $V = \frac{1}{3}\pi r^2 h$ $SA = \pi rl + \pi r^2$

Example 2: A spherical balloon is being deflated at a constant rate of 20 cubic cm per second. How fast is the radius of the balloon changing at the instant when the balloon's radius is 12 cm?

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KNOW: **WANT:**

Step 3: Find an equation relating to the relevant variables.

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Step 5: Substitute in what you **KNOW** from **Step 2** and any information that your equation in **Step 3** can give you and solve for the quantity you WANT**.**

Example 3: A cylindrical tank standing upright (with one circular base on the ground) has a radius of 22 cm for the base. How fast does the water level in the tank drop when the water is being drained at 28 cm^3/sec? Note: The formula right circular cylinder is

$$V = \pi r^2 h.$$

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Example 4: Sand falls from an overhead bin and accumulates in a conical pile with a radius that is always four times its height. Suppose the height of the pile increases at a rate of 3 cm/s when the pile is 15 cm high. At what rate is the sand leaving the bin at that instant? Note: The formula of conical pile (aka cone) is $V = \frac{\pi}{3} r^2 h$.

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Example 5: A rope passing through a capstan on a dock is attached to a boat offshore at water level. The rope is pulled in at a constant rate of 5 ft/s, and the capstan is 4 ft vertically above the water. How fast is the boat traveling when it is 10 ft from the dock?

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Example 6: A hemispherical tank with a radius of 5 m is filled from an inflow pipe at a rate of 4 m^3/min. How fast is the water level rising when the water level is 3 m from the bottom of the tank? (Hint: The volume of a cap of thickness h sliced from a sphere of radius r is $\frac{\pi h^2(3r-h)}{3}$).

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Step 5: Substitute in what you **KNOW** from **Step 2** and any information that your equation in **Step 3** can give you and solve for the quantity you WANT**.**

Example 7: A plane is flying directly away from you at 500 mph at an altitude of 3 miles.

(a) How fast is the plane's distance from you increasing at the moment when the plane is flying over a point on the ground 4 miles from you?

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Step 4: Use implicit differentiation to differentiate the equation with respect to time *t.*

Step 5: Substitute in what you **KNOW** from **Step 2** and any information that your equation in **Step 3** can give you and solve for the quantity you WANT**.**

Example 7: A plane is flying directly away from you at 500 mph at an altitude of 3 miles.
(b) How fast is the angle of elevation changing when it is $\pi/3$?

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Example 8: A ladder 5 meters long rests on horizontal ground and leans against a vertical wall. The foot of the ladder is pulled away from the wall at the rate of 0.3 m/sec. How fast is the top sliding down the wall when the foot of the ladder is 3 m from the wall?

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Example 9: A streetlight fastened to the top of a 20-ft high pole. If a 5-ft tall woman walks away from the pole in a straight line over level ground at a rate of 6 ft/s, how fast is the length of her shadow changing when she is 18 ft away from the pole?

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Example 10: A lighthouse is 375 m from a straight shoreline. Its light rotates 5 times per minute. How fast is the light spot moving along the shore when it hits a point 175 m away from the point nearest the lighthouse?

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