MA 165 LESSONS 17-18: RELATED RATES

Example 1: If x and y are both functions of t and $x + y^3 = 2$.

(a)	Find $\frac{dy}{dy}$ when	$n\frac{dx}{dt} = -2 \text{ and } y = 1.$
(a)	$\frac{1}{dt}$	$\frac{1}{dt} = 2 \text{ and } y = 1.$

(b)	Find $\frac{dx}{dt}$ when $\frac{dy}{dt} = 3$ and $x = 1$.
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Recipe for Solving a Related Rates Problem

- Step 1: Draw a good picture. Label all constant values and give variable names to any changing quantities.
- Step 2: Determine what information you KNOW and what you WANT to find.
- **Step 3:** Find an equation relating the relevant variables. This usually involves a formula from geometry, similar triangles, the Pythagorean Theorem, or a formula from trigonometry. **Use your picture!**
- **Step 4:** Use implicit differentiation to differentiate the equation with respect to time t.
- **Step 5:** Substitute in what you **KNOW** from **Step 2** and any information that your equation in **Step 3** can give you and solve for the quantity you **WANT**. Do **NOT** substitute before this step!

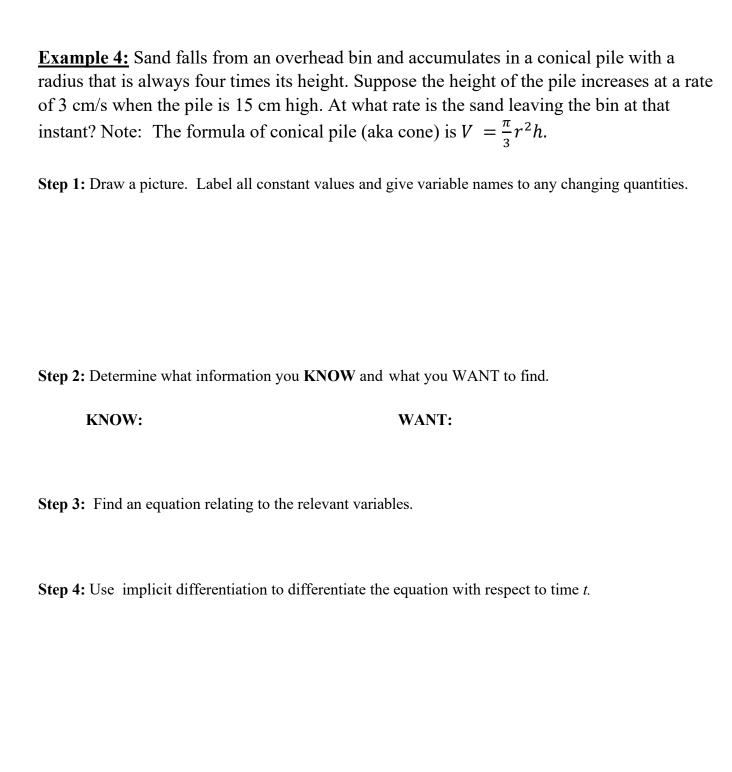
Some Useful Formulas

Right Triangle	Triangle	Trapezoid
Pythagorean Theorem:	$A = \frac{1}{2}bh$	$A = \frac{1}{2}(a+b)h$
$a^2 + b^2 = c^2$	P = a + b + c	
Rectangular Box	Rectangle	Circle
V = lwh	A = lw	$A = \pi r^2$
S = 2(hl + lw + hw)	P = 2l + 2w	$C=2\pi r$
Right Circular Cylinder $V = \pi r^2 h$	$\frac{\text{Sphere}}{V = \frac{4}{3}\pi r^3}$	Cone 1
$SA = 2\pi rh$	$S = 4\pi r^2$	$V = \frac{1}{3}\pi r^2 h$ $SA = \pi r l + \pi r^2$

Example 2: A spherical balloon is being deflated second. How fast is the radius of the balloon changradius is 12 cm?	1
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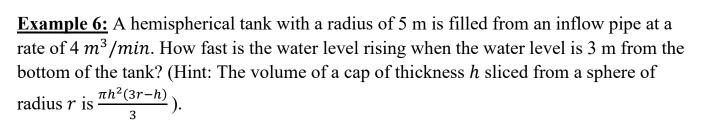
Example 3: A cylindrical tank standing upright (with one circular base on the ground) has a radius of 22 cm for the base. How fast does the water level in the tank drop when the water is being drained at 28 cm ³ /sec? Note: The formula right circular cylinder is $V = \pi r^2 h$.
Step 1: Draw a picture. Label all constant values and give variable names to any changing quantities.
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Step 5: Substitute in what you KNOW from Step 2 and any information that your equation in Step 3 can give you and solve for the quantity you WANT.



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Example 5: A rope passing through a capstan of water level. The rope is pulled in at a constant above the water. How fast is the boat traveling	rate of 5 ft/s, and the capstan is 4 ft vertically
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Example 7: A plane is flying directly away from you at 500 mph at an altitude of 3 miles. (a) How fast is the plane's distance from you increasing at the moment when the plane is flying over a point on the ground 4 miles from you?
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Example 8: A ladder 5 meters long rests on horizontal ground and leans against a vertical wall. The foot of the ladder is pulled away from the wall at the rate of 0.3 m/sec. How fast is the top sliding down the wall when the foot of the ladder is 3 m from the wall?
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Example 9: A streetlight fastened to the top of a 20-ft high pole. If a 5-ft tall woman walks away from the pole in a straight line over level ground at a rate of 6 ft/s, how fast is the length of her shadow changing when she is 18 ft away from the pole?
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Example 10: A lighthouse is 375 m from a straight shoreline. Its light rotates 5 times per minute. How fast is the light spot moving along the shore when it hits a point 175 m away
from the point nearest the lighthouse?
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