Lesson 17: Maxima and Minima

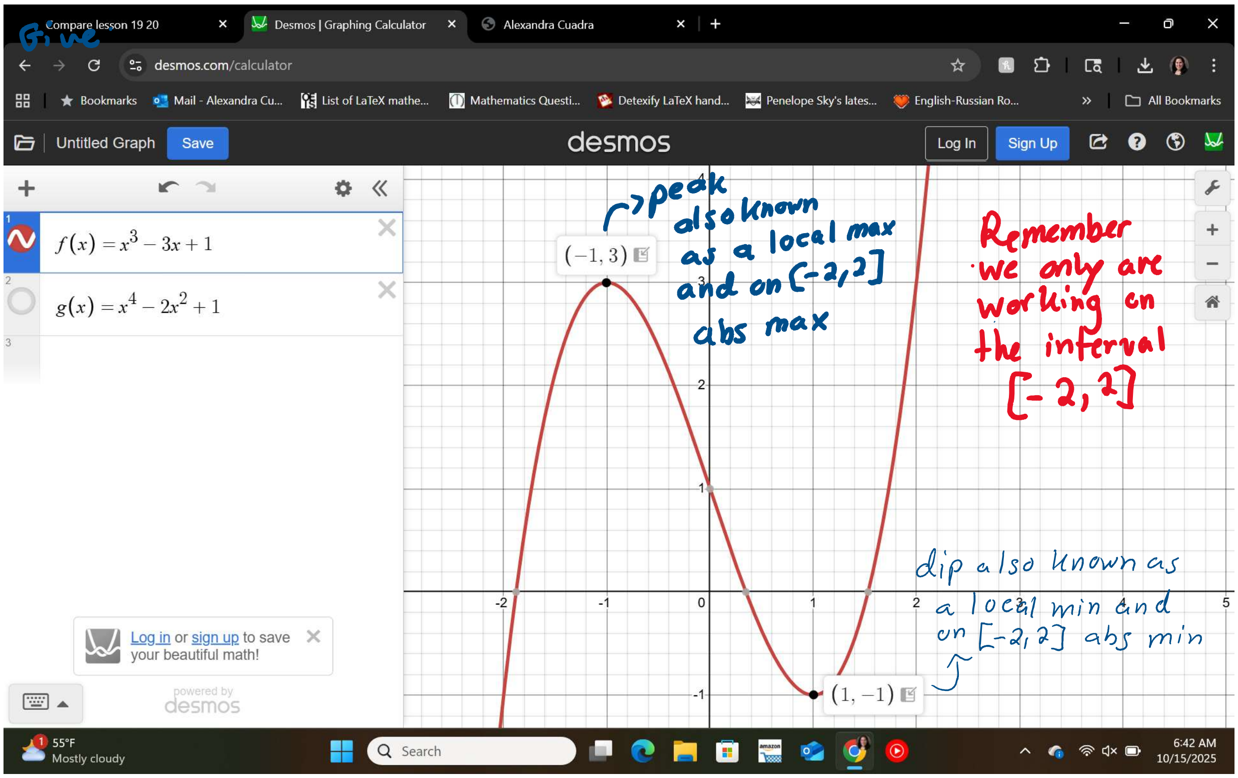
Wednesday, October 15, 2025 6:42 AM

Let's start w/ a Desmos Demo

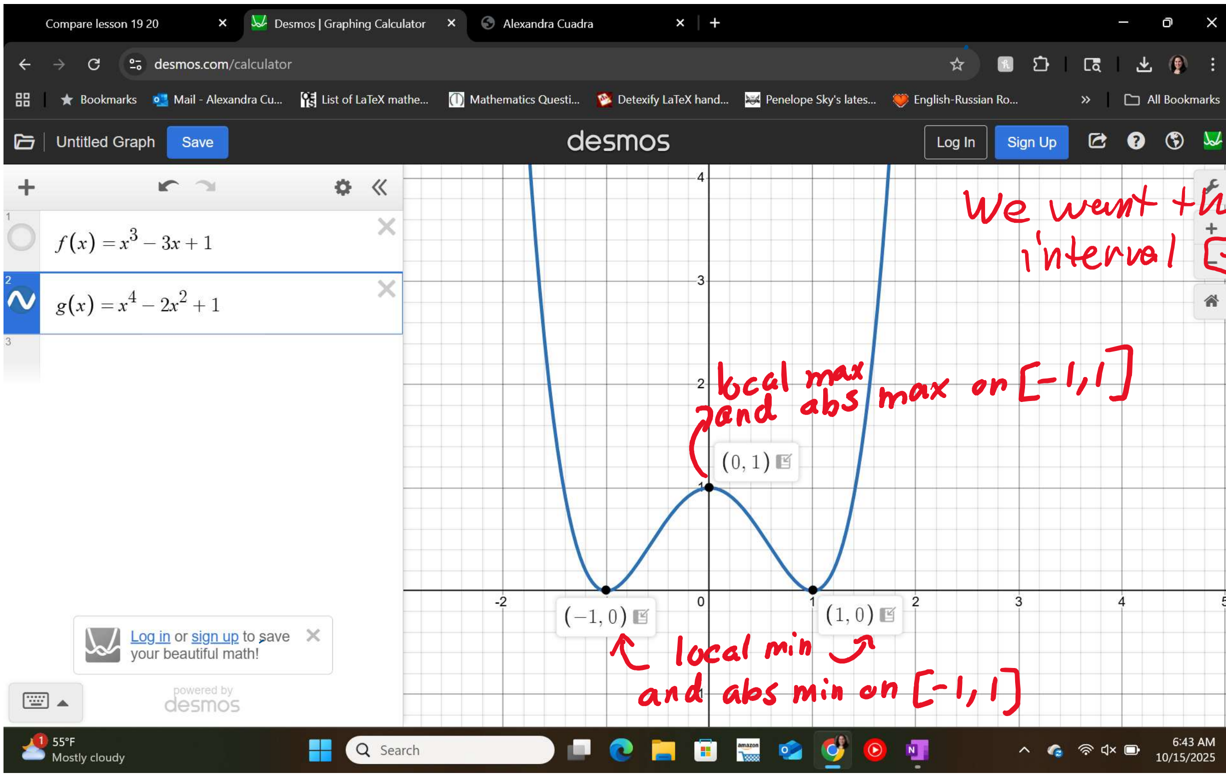
Let $f(x) = x^3 - 3x + 1$

What do you notice about the graph on $[-2, 2]$?

Where does the function "peak" or "dip"?



Similarly if we have $f(x) = x^4 - 2x^2 + 1$ on $[-1, 1]$. Then



So based on both pictures, we can have local/absolute maxima or minima in the interval or at the endpoints.

Definitions:

Term	Informal Description	Formal
Local (Relative) Max	Highest nearby value (small "hill")	$f(c) \geq f(x)$ for all x near c.
Local (Relative) Min	Lowest nearby value (small "valley")	$f(c) \leq f(x)$ for all x near c.
Absolute Max	Highest value on the entire interval	$f(c) \geq f(x)$ for all x in $[a,b]$
Absolute Min	Lowest value on the entire interval	$f(c) \leq f(x)$ for all x in $[a,b]$

Note: Endpoints can be absolute extrema but not local extrema

Extrema Value Theorem (EVT)

If f is continuous on a closed interval $[a,b]$, then f has both an absolute max and an absolute min on that interval.

Now that we know these maxima and minima must exist when a function is continuous on $[a,b]$, how do we find them?

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Recipe

1. Find all points when $f'(x)=0$ or $f'(x)$ DNE
2. Evaluate points from 1 and endpoints in $f(x)$.
3. Compare the values to locate absolute max/min.

Ex 1: Find the abs extrema of

$$y = x^4 - 2x^3 \text{ on } [-1,1]$$

Step 1: Find x when $f'(x)=0$.

$$f'(x) = 4x^3 - 2(3)x^2$$
$$= 4x^3 - 6x^2$$

Set equal to 0. $4x^3 - 6x^2 = 0$

$$2x^2(2x-3)=0$$

$2x^2=0$ $\quad$ $2x-3=0$

$x=0$ $\quad$ $\cancel{x=3/2}$ b/c not in the interval

Step 2+3:

x	$f(x)=x^4-2x^3$	Conclusion
-1	$1-2(-1)=3$	abs max
0	0	
1	$1-2=-1$	abs min

Table includes values from Step 1 and endpoints $[-1,1]$

Ex 2: Find the abs extrema of

$$y = xe^x \text{ on } [-2,0]$$

Step 1: Find x where $f'(x)=0$.

$u = x$ $\quad$ $v = e^x$

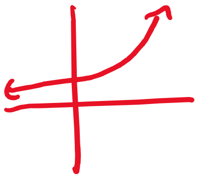
$u' = 1$ $\quad$ $v' = e^x$

$u = x \qquad v = e^x$

$u' = 1 \qquad v' = e^x$

$$y' = 1 \cdot e^x + xe^x = 0$$

$$e^x(1+x) = 0$$



$e^x = 0$ NEVER $\qquad 1 + x = 0$

$x = -1$ Check to see if $[-2, 0]$ ✓

Step 2+3:

x	$f(x) = xe^x$	Conclusion
-2	$-2e^{-2} = -2/e^2$	
-1	$-e^{-1} = -1/e = -e/e^2$	Abs min
0	0	Abs max

Ex 3: Find the abs extrema of

$$y = \frac{6x^2}{x+1} \text{ on } [0, 5]$$

Step 1: Find x when $f'(x) = 0$ and $f'(x)$ DNE

$u = 6x^2 \qquad v = x + 1$

$u' = 12x \qquad v' = 1$

$$y' = \frac{u'v - uv'}{v^2} = \frac{(12x)(x+1) - 6x^2(1)}{(x+1)^2}$$

$$= \frac{12x^2 + 12x - 6x^2}{(x+1)^2}$$

$$= \frac{6x^2 + 12x}{(x+1)^2}$$

$f'(x) = 0$: $6x^2 + 12x = 0$ $\qquad$ $f'(x)$ DNE: $(x+1)^2 = 0$

$6x(x+2) = 0$ $\qquad$ $x + 1 = 0$

$\underline{f'(x)=0}$: $6x^2+12x=0$

$6x(x+2)=0$

$x=0,\ x=-2$

$\underline{f'(x) \text{ DNE}}$: $(x+1)^2=0$

$x+1=0$

$x=-1$

$\underline{\text{Check}}$: If $x=0,-1,-2$ in $[0,5]$? Only $\underline{x=0}$

$\underline{\text{Step } 2+3}$:

x	$f(x)=\frac{6x^2}{x+1}$	Conclusion
0	$\frac{0}{1}=0$	Abs min
5	$\frac{\cancel{6}\cdot 5^2}{\cancel{5+1}}=25$	Abs max