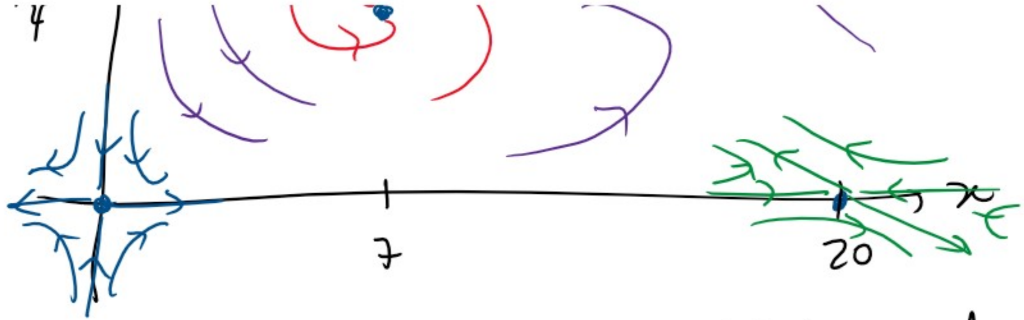


5. Interpret in terms of rabbits and foxes

as $t \to \infty$ $\quad x \longrightarrow 7$ (rabbits)

$y \longrightarrow \frac{13}{4}$ (foxes)

populations converge to a single value
"coexistence"

II. Competitors:

$x(t)$ - rabbits $\qquad y(t)$ - deer

- both eat vegetation
- neither preys on the other

$\Big\}$ competition model

Equations:

$$\frac{dx}{dt} = \underbrace{a_1 x - b_1 x^2}_{\text{logistic growth}} \; \underbrace{- \; c_1 xy}_{\text{competition for resources}}$$

$$\frac{dy}{dt} = a_2 y - b_2 y^2 - c_2 xy$$

logistic growth

competition for resources (both negative)

Note: $a_i, b_i, c_i > 0$

Rewrite: $\quad x' = x(a_1 - b_1 x - c_1 y)$

$y' = y(a_2 - b_2 y - c_2 x)$

This system has four critical points:

$\rightarrow (0,0)$

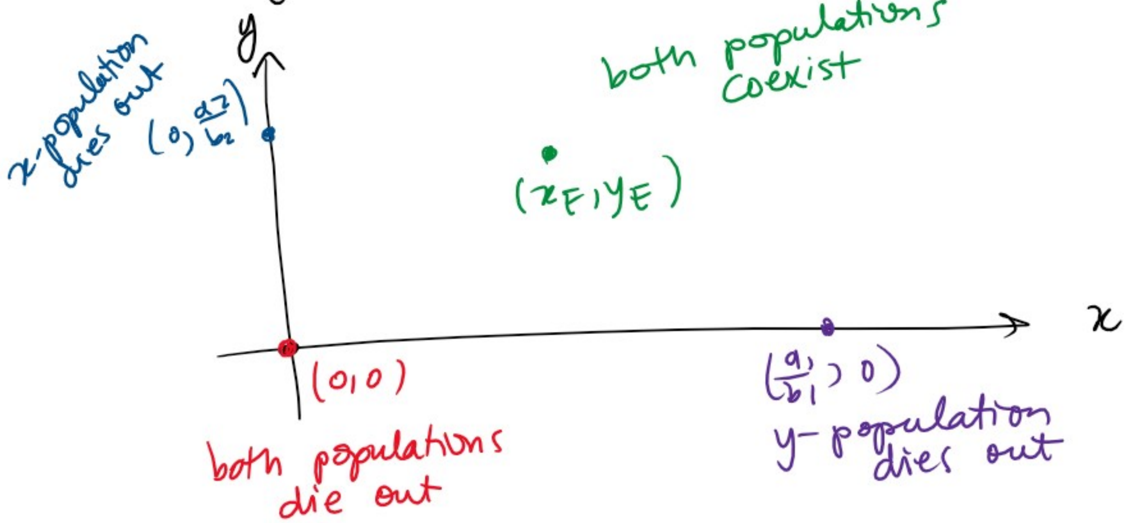
This system has $\underline{\text{four}}$ critical p...

$x=0$, and $y=0 \rightarrow (0,0)$

if $x=0 \rightarrow a_2 - b_2 y = 0 \longrightarrow \left(0, \frac{a_2}{b_2}\right)$

if $y=0 \rightarrow a_1 - b_1 x = 0 \longrightarrow \left(\frac{a_1}{b_1}, 0\right)$

if $\begin{cases} a_1 - b_1 x - c_1 y = 0 \\ a_2 - b_2 y - c_2 x = 0 \end{cases}$ call the solution (x_E, y_E)



Often <u>coexistence</u> is a goal.

<u>WANT</u>: (x_E, y_E) to be <u>stable</u>

<u>Coexistence Criteria:</u>

If $\underbrace{c_1 c_2}_{\text{Competition}} < \underbrace{b_1 b_2}_{\text{inhibition}}$ then (x_E, y_E) is an asymptotically stable critical point

then the 2 species coexist.

← to calculate $\underline{\underline{J}}$

<u>Ex</u>: $x' = 30x - 3x^2 + xy = x(30 - 3x + y)$

$y' = 60y - 3y^2 + 4xy = y(60 - 3y + 4x)$

Find the critical point for coexistence: (x_E, y_E)

$$30 - 3x + y = 0$$

$$y = 3x - 30$$

$$60 - 3y + 4x = 0$$

$$60 - 3(3x - 30) + 4x = 0$$

$$60 - 9x + 90 + 4x = 0$$

$$150 = 5x$$

$$x = 30$$

$$y = 3(30) - 30$$

$$y = 60$$

$$\boxed{(x_E, y_E) = (30, 60)}$$

WANT: $(30, 60)$ to be stable or asymptotically stable

@ $(30, 60)$ $\quad \underline{J} = \begin{bmatrix} -90 & 30 \\ 240 & -180 \end{bmatrix}$ $\quad \lambda = -15(9 \pm \sqrt{41})$

$$\lambda < 0$$

improper nodal sink

asymptotically stable

$\rightarrow$ coexistence