

No

- ① $y^{(n)} = f(x)$ integrate for n times
- ② 2nd order. Let $u = y'$

Yes

Write the DE in the form

(*) $M(x,y)dx + N(x,y)dy$

This is always possible by simply multiplying both sides of the DE by dx



Yes

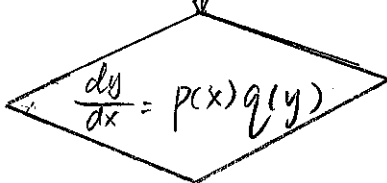
Find out $\phi(x,y) = \int M(x,y)dx + h(y)$.
Then $\phi = c$ is the general solution.

No

Rewrite the DE in the form

(***) $\frac{dy}{dx} = f(x,y)$

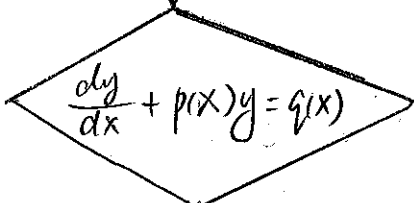
This is always possible. Indeed, from (*) we have $\frac{dy}{dx} = -\frac{M(x,y)}{N(x,y)}$.



Yes

separation of variables: $\int \frac{dy}{q(y)} = \int p(x)dx + C$

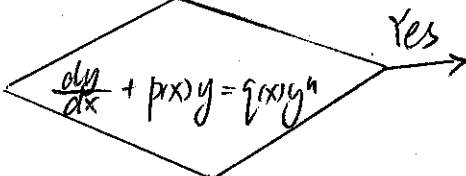
No



Yes

$I(x) = e^{\int p(x)dx} \Rightarrow (I(x)y)' = I(x)q(x)$

No



Yes

Divided by y^n . Let $u = y^{1-n}$. Then $u' + (1-n)p(x)u = (1-n)q(x)$

No

No. (Go back to (**))

$$f(x, y) = f(tx, ty)$$

Yes

$$\text{Let } V = \frac{y}{x} \Rightarrow xV' + V = f(1, V)$$

No (Go back to (*))

(i) If $\frac{\partial_y M - \partial_x N}{N}$ depends only on x , then

$$I(x) = e^{\int \frac{\partial_y M - \partial_x N}{N} dx}$$

is an integrating factor

(ii) If $\frac{\partial_y M - \partial_x N}{M}$ depends only on y , then

$$I(y) = e^{-\int \frac{\partial_y M - \partial_x N}{M} dy}$$

is an integrating factor

Exer: determine the type of the following DE.

$$(a) (x^4 - 2t^3x) dt + (t^4 - 2tx^3) dx = 0$$

$$(b) \sec^2 t \tan x dt + \sec^2 x \tan t dx = 0$$

$$(c) x^2 + t^2 \frac{dx}{dt} = tx \frac{dx}{dt}$$

$$(d) \frac{dx}{dt} = -\frac{x}{t+x^3}$$

$$(e) x dy - 4y dx = x \sqrt{y} dx$$

$$(f) \left(\frac{1}{x} \sin \frac{t}{x} - \frac{x}{t^2} \cos \frac{x}{t} + 1 \right) dt + \left(\frac{1}{t} \cos \frac{x}{t} - \frac{t}{x^2} \sin \frac{t}{x} + \frac{1}{x^2} \right) dx = 0$$

$$(g) dy + \frac{xy}{1+x^2} dx = \frac{1}{x(1+x^2)} dx$$

Solution:

- (a) homogeneous
- (b) separable
- (c) homogeneous
- (d) exact equation
- (e) Bernoulli equation
- (f) exact equation
- (g) first order linear equation.

Question: What is the type of

$$\frac{dx}{dt} = \frac{x}{t+x^3}$$