1. b. c. e

a. counter example $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

d. counter example $A = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$

2. (i) (ii) (iii)

$\{v_1, v_2\}$ is orthogonal, but might not be orthonormal

3.

$$p(\lambda) = \begin{vmatrix} \lambda-4 & -1 & 1 \\ -2 & \lambda-5 & 2 \\ -1 & -1 & \lambda-2 \end{vmatrix} = \begin{vmatrix} \lambda-4 & -1 & 0 \\ -2 & \lambda-5 & \lambda-3 \\ -1 & -1 & \lambda-3 \end{vmatrix}$$

$$= (\lambda-4) \begin{vmatrix} \lambda-5 & \lambda-3 \\ -1 & \lambda-3 \end{vmatrix} + 1 \begin{vmatrix} -2 & \lambda-3 \\ -1 & \lambda-3 \end{vmatrix}$$

$$= (\lambda-3)\left[(\lambda-4)(\lambda-4) - 1\right] = (\lambda-3)(\lambda^2 - 8\lambda + 15)$$

$$= (\lambda-3)^2(\lambda-5)$$

$\lambda_1 = 3$

$$\lambda_1 I_3 - A \sim \begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad v_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$\lambda_2 = 5$

$$\lambda_2 I_3 - A \sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix} \qquad v_3 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \qquad P = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix}$$