MA26200: EXAM II **NAME:** ____________________

1. The eigenvalues of the matrix $A = \begin{bmatrix} 2 & 4 \\ 3 & 3 \end{bmatrix}$ are

A. 2, 3

B. 3, 4

C. -2, -3

D. -1, 6

E. 1, 4

2. Find all the eigenvalues and eigenspaces of $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$.

3. The subspace of $\mathbb{R}^3$ spanned by $\{(1,2,3),(1,3,5),(1,5,k)\}$ has dimension 3 if

A. $k \neq 1$

B. $k \neq 9$

C. $k = 0$

D. $k = 1$

E. $k = 9$

4. The subspace of $\mathbb{P}_3$, the space of all polynomials of degree no more than 3, spanned by $\{1, x-1, (x-1)^2, (x-1)^3\}$ has dimension

A. 0

B. 1

C. 2

D. 3

E. 4

5. Find a basis for the rowspace(A), where

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 1 & -2 \\ 3 & 6 & 3 & -7 \end{bmatrix}.$$

6. $T : \mathbb{R}^6 \to \mathbb{R}^5$ is given by $x \mapsto Ax$, where

$$A = \begin{bmatrix} 1 & 2 & 1 & 3 & 1 & 2 \\ 2 & 5 & 5 & 6 & 4 & 5 \\ 3 & 7 & 6 & 11 & 6 & 9 \\ 1 & 5 & 10 & 8 & 9 & 9 \\ 2 & 6 & 8 & 11 & 9 & 12 \end{bmatrix}.$$

C_i denotes the i-th column of A. Then a basis of $\text{Rng}(T)$ consists of

A. $\{C_1, C_2, C_4\}$

B. $\{C_1, C_2, C_3, C_4\}$

C. $\{C_1, C_2, C_3\}$

D. $\{C_4, C_5\}$

E. $\{C_1, C_2, C_3, C_4, C_5\}$

7. $T : \mathbb{R}^5 \to \mathbb{R}^5$ is given by $x \mapsto Ax$, where

$$A = \begin{bmatrix} 1 & 1 & 3 & 1 & 5 \\ 2 & 3 & 8 & 4 & 13 \\ 1 & 3 & 7 & 6 & 13 \\ 3 & 5 & 13 & 9 & 25 \\ 2 & 3 & 8 & 7 & 19 \end{bmatrix}.$$

Find a basis for $\mathrm{Ker}(T)$.

8. Find the general solution to the differential equation

$$y'' + y = e^{2x}\sin x - x.$$

9. Suppose that A is an $n \times n$ square matrix. Which of the following statements must be equivalent to the fact rowspace$(A) = \mathbb{R}^n$

a. A is non-defective.

b. A is invertible.

c. $Ax = b$ is consistent for any b in $\mathbb{R}^n$.

d. $Ax = 0$ is consistent.

e. Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be the linear transformation defined by $x \mapsto Ax$. Then $\mathrm{Rng}(T) = \mathbb{R}^n$.

A. Only b, c and e.

B. Only a, b, c and e.

C. Only a, b and c.

D. None of the above statements.

E. All of the above statements.

10. An 4×4 matrix A has eigenvalues $1, -1, 2, 4$. Which of the following must be true?

a. A is invertible.

b. A is non-defective.

c. A has 4 linearly independent eigenvectors.

d. Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be the linear transformation defined by $x \mapsto Ax$. Then $\mathrm{Ker}(T) = \{0\}$.

A. Only b, c and d.

B. Only b.

C. Only c.

D. None of the above statements.

E. All of the above statements.

11. Find the general solution to the differential equation

$$(D^2 + 4D + 5)^2(D - 1)^3(D + 2)y = 0.$$